CHAPTER 5

SIGNAL ENCODING TECHNIQUES

Even the natives have difficulty mastering this peculiar vocabulary.

—The Golden Bough, Sir James George Frazer

KEY POINTS

- Both analog and digital information can be encoded as either analog or digital signals. The particular encoding that is chosen depends on the specific requirements to be met and the media and communications facilities available.
- **Digital data, digital signals:** The simplest form of digital encoding of digital data is to assign one voltage level to binary one and another to binary zero. More complex encoding schemes are used to improve performance, by altering the spectrum of the signal and providing synchronization capability.
- **Digital data, analog signal:** A modem converts digital data to an analog signal so that it can be transmitted over an analog line. The basic techniques are amplitude shift keying (ASK), frequency shift keying (FSK), and phase shift keying (PSK). All involve altering one or more characteristics of a carrier frequency to represent binary data.
- **Analog data, digital signals:** Analog data, such as voice and video, are often digitized to be able to use digital transmission facilities. The simplest technique is pulse code modulation (PCM), which involves sampling the analog data periodically and quantizing the samples.
- **Analog data, analog signals:** Analog data are modulated by a carrier frequency to produce an analog signal in a different frequency band, which can be utilized on an analog transmission system. The basic techniques are amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM).

In Chapter 3 a distinction was made between analog and digital data and analog and digital signals. Figure 3.14 suggested that either form of data could be encoded into either form of signal.

Figure 5.1 is another depiction that emphasizes the process involved. For **digital signaling**, a data source $g(t)$, which may be either digital or analog, is encoded into a digital signal $x(t)$. The actual form of $x(t)$ depends on the encoding technique and is chosen to optimize use of the transmission medium. For example, the encoding may be chosen to conserve bandwidth or to minimize errors.

The basis for **analog signaling** is a continuous constant-frequency signal known as the **carrier signal**. The frequency of the carrier signal is chosen to be compatible with the transmission medium being used. Data may be transmitted using a carrier signal by modulation. **Modulation** is the process of encoding

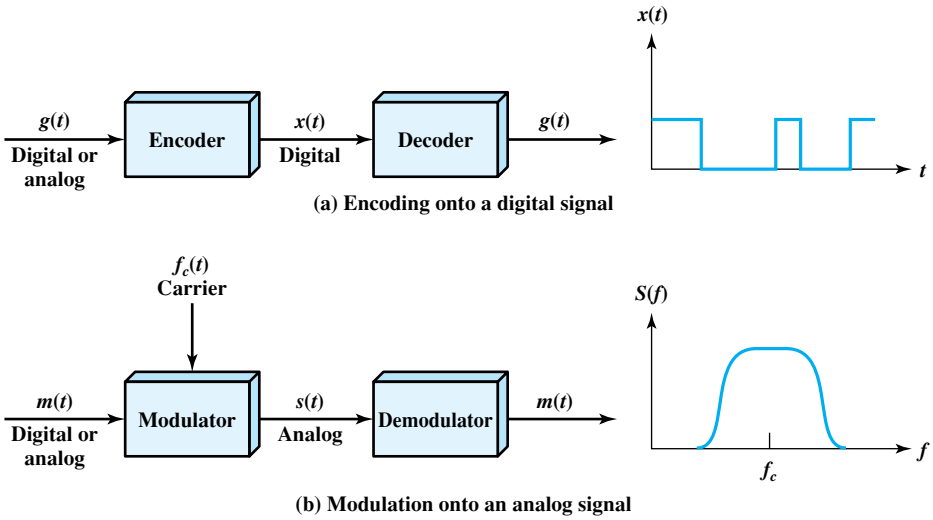

Figure 5.1 Encoding and Modulation Techniques

source data onto a carrier signal with frequency f_c. All modulation techniques involve operation on one or more of the three fundamental frequency domain parameters: amplitude, frequency, and phase.

The input signal $m(t)$ may be analog or digital and is called the modulating signal or **baseband signal**. The result of modulating the carrier signal is called the modulated signal $s(t)$. As Figure 5.1b indicates, $s(t)$ is a bandlimited (bandpass) signal. The location of the bandwidth on the spectrum is related to f_c and is often centered on f_c. Again, the actual form of the encoding is chosen to optimize some characteristic of the transmission.

Each of the four possible combinations depicted in Figure 5.1 is in widespread use. The reasons for choosing a particular combination for any given communication task vary. We list here some representative reasons:

- **Digital data, digital signal:** In general, the equipment for encoding digital data into a digital signal is less complex and less expensive than digital-to-analog modulation equipment.
- **Analog data, digital signal:** Conversion of analog data to digital form permits the use of modern digital transmission and switching equipment. The advantages of the digital approach were outlined in Section 3.2.
- **Digital data, analog signal:** Some transmission media, such as optical fiber and unguided media, will only propagate analog signals.
- **Analog data, analog signal:** Analog data in electrical form can be transmitted as baseband signals easily and cheaply. This is done with voice transmission over voice-grade lines. One common use of modulation is to shift the bandwidth of a baseband signal to another portion of the spectrum. In this way multiple signals, each at a different position on the

spectrum, can share the same transmission medium. This is known as frequency division multiplexing.

We now examine the techniques involved in each of these four combinations.

5.1 DIGITAL DATA, DIGITAL SIGNALS

A digital signal is a sequence of discrete, discontinuous voltage pulses. Each pulse is a signal element. Binary data are transmitted by encoding each data bit into signal elements. In the simplest case, there is a one-to-one correspondence between bits and signal elements. An example is shown in Figure 3.16, in which binary 1 is represented by a lower voltage level and binary 0 by a higher voltage level. We show in this section that a variety of other encoding schemes are also used.

First, we define some terms. If the signal elements all have the same algebraic sign, that is, all positive or negative, then the signal is **unipolar**. In **polar** signaling, one logic state is represented by a positive voltage level, and the other by a negative voltage level. The **data signaling rate**, or just **data rate**, of a signal is the rate, in bits per second, that data are transmitted. The duration or length of a bit is the amount of time it takes for the transmitter to emit the bit; for a data rate R, the bit duration is $1/R$. The **modulation rate**, in contrast, is the rate at which the signal level is changed. This will depend on the nature of the digital encoding, as explained later. The modulation rate is expressed in baud, which means signal elements per second. Finally, the terms mark and space, for historical reasons, refer to the binary digits 1 and 0, respectively. Table 5.1 summarizes key terms; these should be clearer when we see an example later in this section.

The tasks involved in interpreting digital signals at the receiver can be summarized by again referring to Figure 3.16. First, the receiver must know the timing of each bit. That is, the receiver must know with some accuracy when a bit begins and ends. Second, the receiver must determine whether the signal level for each bit position is high (0) or low (1). In Figure 3.16, these tasks are performed by sampling each bit position in the middle of the interval and comparing the value to a threshold. Because of noise and other impairments, there will be errors, as shown.

What factors determine how successful the receiver will be in interpreting the incoming signal? We saw in Chapter 3 that three factors are important: the

Table 5.1 Key Data Transmission Terms

Term	Units	Definition
Data element	Bits	A single binary one or zero
Data rate	Bits per second (bps)	The rate at which data elements are transmitted
Signal element	Digital: a voltage pulse of constant amplitude	That part of a signal that occupies the shortest interval of a signaling code
	Analog: a pulse of constant frequency, phase, and amplitude	
Signaling rate or modulation rate	Signal elements per second (baud)	The rate at which signal elements are transmitted

Table 5.2 Definition of Digital Signal Encoding Formats

Nonreturn to Zero-Level (NRZ-L)
0 = high level
1 = low level
Nonreturn to Zero Inverted (NRZI)
0 = no transition at beginning of interval (one bit time)
1 = transition at beginning of interval
Bipolar-AMI
0 = no line signal
1 = positive or negative level, alternating for successive ones
Pseudoternary
0 = positive or negative level, alternating for successive zeros
1 = no line signal
Manchester
0 = transition from high to low in middle of interval
1 = transition from low to high in middle of interval
Differential Manchester
Always a transition in middle of interval
0 = transition at beginning of interval
1 = no transition at beginning of interval
B8ZS
Same as bipolar AMI, except that any string of eight zeros is replaced by a string with two code violations
HDB3
Same as bipolar AMI, except that any string of four zeros is replaced by a string with one code violation

signal-to-noise ratio, the data rate, and the bandwidth. With other factors held constant, the following statements are true:

- An increase in data rate increases bit error rate (BER).[1]
- An increase in SNR decreases bit error rate.
- An increase in bandwidth allows an increase in data rate.

There is another factor that can be used to improve performance, and that is the encoding scheme. The encoding scheme is simply the mapping from data bits to signal elements. A variety of approaches have been tried. In what follows, we describe some of the more common ones; they are defined in Table 5.2 and depicted in Figure 5.2.

Before describing these techniques, let us consider the following ways of evaluating or comparing the various techniques.

[1]The BER is the most common measure of error performance on a data circuit and is defined as the probability that a bit is received in error. It is also called the *bit error ratio*. This latter term is clearer, because the term *rate* typically refers to some quantity that varies with time. Unfortunately, most books and standards documents refer to the R in BER as *rate*.

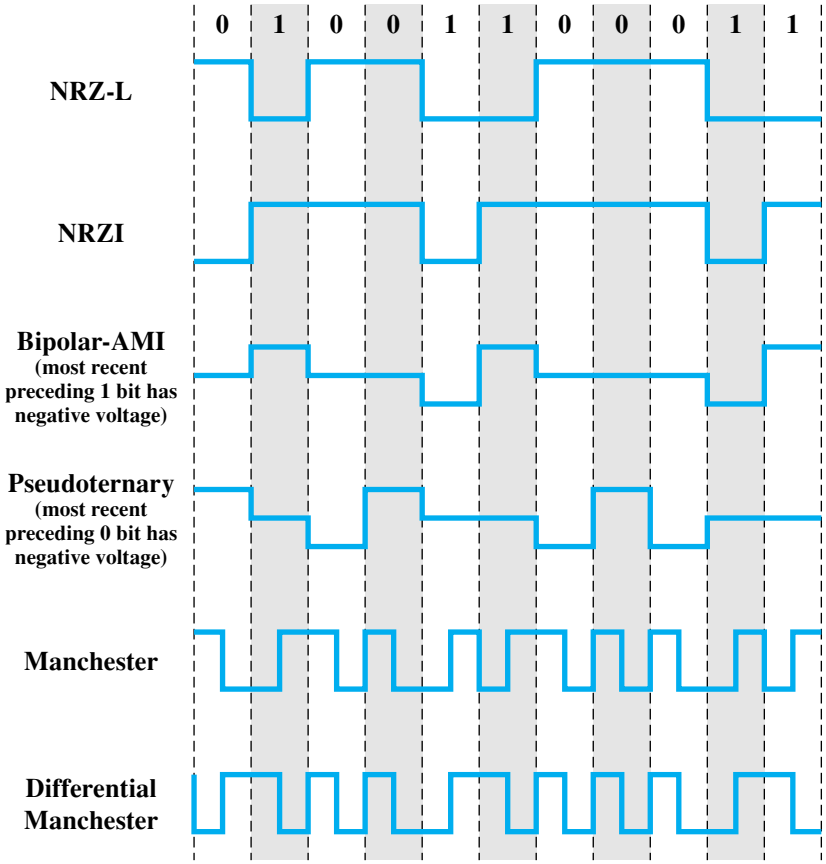

Figure 5.2 Digital Signal Encoding Formats

- **Signal spectrum:** Several aspects of the signal spectrum are important. A lack of high-frequency components means that less bandwidth is required for transmission. In addition, lack of a direct-current (dc) component is also desirable. With a dc component to the signal, there must be direct physical attachment of transmission components. With no dc component, ac coupling via transformer is possible; this provides excellent electrical isolation, reducing interference. Finally, the magnitude of the effects of signal distortion and interference depend on the spectral properties of the transmitted signal. In practice, it usually happens that the transmission characteristics of a channel are worse near the band edges. Therefore, a good signal design should concentrate the transmitted power in the middle of the transmission bandwidth. In such a case, a smaller distortion should be present in the received signal. To meet this objective, codes can be designed with the aim of shaping the spectrum of the transmitted signal.
- **Clocking:** We mentioned the need to determine the beginning and end of each bit position. This is no easy task. One rather expensive approach is to provide

a separate clock lead to synchronize the transmitter and receiver. The alternative is to provide some synchronization mechanism that is based on the transmitted signal. This can be achieved with suitable encoding, as explained subsequently.

- **Error detection:** We will discuss various error-detection techniques in Chapter 6 and show that these are the responsibility of a layer of logic above the signaling level that is known as data link control. However, it is useful to have some error detection capability built into the physical signaling encoding scheme. This permits errors to be detected more quickly.
- **Signal interference and noise immunity:** Certain codes exhibit superior performance in the presence of noise. Performance is usually expressed in terms of a BER.
- **Cost and complexity:** Although digital logic continues to drop in price, this factor should not be ignored. In particular, the higher the signaling rate to achieve a given data rate, the greater the cost. We shall see that some codes require a signaling rate that is greater than the actual data rate.

We now turn to a discussion of various techniques.

Nonreturn to Zero (NRZ)

The most common, and easiest, way to transmit digital signals is to use two different voltage levels for the two binary digits. Codes that follow this strategy share the property that the voltage level is constant during a bit interval; there is no transition (no return to a zero voltage level). For example, the absence of voltage can be used to represent binary 0, with a constant positive voltage used to represent binary 1. More commonly, a negative voltage represents one binary value and a positive voltage represents the other. This latter code, known as **Nonreturn to Zero-Level** (NRZ-L), is illustrated[2] in Figure 5.2. NRZ-L is typically the code used to generate or interpret digital data by terminals and other devices. If a different code is to be used for transmission, it is generated from an NRZ-L signal by the transmission system [in terms of Figure 5.1, NRZ-L is $g(t)$ and the encoded signal is $x(t)$].

A variation of NRZ is known as **NRZI** (Nonreturn to Zero, invert on ones). As with NRZ-L, NRZI maintains a constant voltage pulse for the duration of a bit time. The data themselves are encoded as the presence or absence of a signal transition at the beginning of the bit time. A transition (low to high or high to low) at the beginning of a bit time denotes a binary 1 for that bit time; no transition indicates a binary 0.

NRZI is an example of **differential encoding**. In differential encoding, the information to be transmitted is represented in terms of the changes between successive signal elements rather than the signal elements themselves. The encoding of the current bit is determined as follows: If the current bit is a binary 0, then the

[2]In this figure, a negative voltage is equated with binary 1 and a positive voltage with binary 0. This is the opposite of the definition used in virtually all other textbooks. The definition here conforms to the use of NRZ-L in data communications interfaces and the standards that govern those interfaces.

current bit is encoded with the same signal as the preceding bit; if the current bit is a binary 1, then the current bit is encoded with a different signal than the preceding bit. One benefit of differential encoding is that it may be more reliable to detect a transition in the presence of noise than to compare a value to a threshold. Another benefit is that with a complex transmission layout, it is easy to lose the sense of the polarity of the signal. For example, on a multidrop twisted-pair line, if the leads from an attached device to the twisted pair are accidentally inverted, all 1s and 0s for NRZ-L will be inverted. This does not happen with differential encoding.

The NRZ codes are the easiest to engineer and, in addition, make efficient use of bandwidth. This latter property is illustrated in Figure 5.3, which compares the spectral density of various encoding schemes. In the figure, frequency is normalized to the data rate. Most of the energy in NRZ and NRZI signals is between dc and half the bit rate. For example, if an NRZ code is used to generate a signal with data rate of 9600 bps, most of the energy in the signal is concentrated between dc and 4800 Hz.

The main limitations of NRZ signals are the presence of a dc component and the lack of synchronization capability. To picture the latter problem, consider that with a long string of 1s or 0s for NRZ-L or a long string of 0s for NRZI, the output is a constant voltage over a long period of time. Under these circumstances, any drift between the clocks of transmitter and receiver will result in loss of synchronization between the two.

Because of their simplicity and relatively low frequency response characteristics, NRZ codes are commonly used for digital magnetic recording. However, their limitations make these codes unattractive for signal transmission applications.

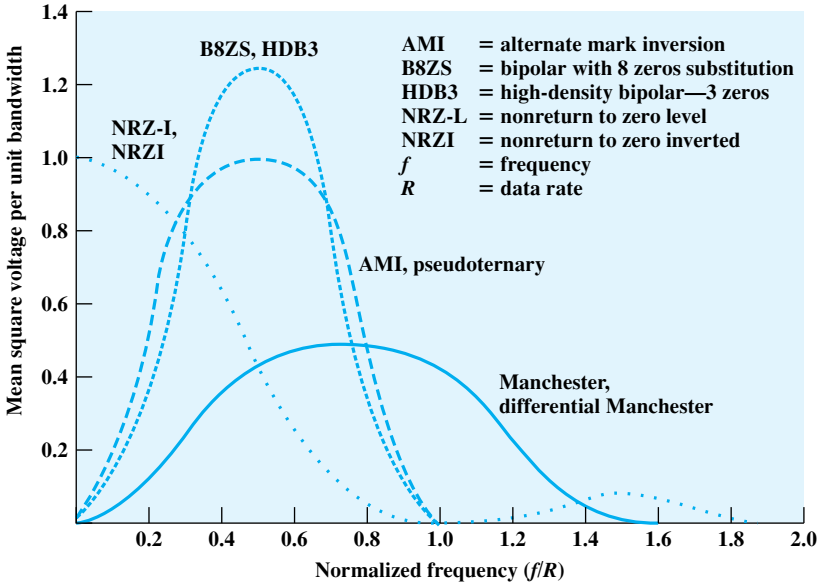

Figure 5.3 Spectral Density of Various Signal Encoding Schemes

Multilevel Binary

A category of encoding techniques known as multilevel binary addresses some of the deficiencies of the NRZ codes. These codes use more than two signal levels. Two examples of this scheme are illustrated in Figure 5.2, bipolar-AMI (alternate mark inversion) and pseudoternary.[3]

In the case of the **bipolar-AMI** scheme, a binary 0 is represented by no line signal, and a binary 1 is represented by a positive or negative pulse. The binary 1 pulses must alternate in polarity. There are several advantages to this approach. First, there will be no loss of synchronization if a long string of 1s occurs. Each 1 introduces a transition, and the receiver can resynchronize on that transition. A long string of 0s would still be a problem. Second, because the 1 signals alternate in voltage from positive to negative, there is no net dc component. Also, the bandwidth of the resulting signal is considerably less than the bandwidth for NRZ (Figure 5.3). Finally, the pulse alternation property provides a simple means of error detection. Any isolated error, whether it deletes a pulse or adds a pulse, causes a violation of this property.

The comments of the previous paragraph also apply to **pseudoternary**. In this case, it is the binary 1 that is represented by the absence of a line signal, and the binary 0 by alternating positive and negative pulses. There is no particular advantage of one technique versus the other, and each is the basis of some applications.

Although a degree of synchronization is provided with these codes, a long string of 0s in the case of AMI or 1s in the case of pseudoternary still presents a problem. Several techniques have been used to address this deficiency. One approach is to insert additional bits that force transitions. This technique is used in ISDN (integrated services digital network) for relatively low data rate transmission. Of course, at a high data rate, this scheme is expensive, because it results in an increase in an already high signal transmission rate. To deal with this problem at high data rates, a technique that involves scrambling the data is used. We examine two examples of this technique later in this section.

Thus, with suitable modification, multilevel binary schemes overcome the problems of NRZ codes. Of course, as with any engineering design decision, there is a tradeoff. With multilevel binary coding, the line signal may take on one of three levels, but each signal element, which could represent $\log_2 3 = 1.58$ bits of information, bears only one bit of information. Thus multilevel binary is not as efficient as NRZ coding. Another way to state this is that the receiver of multilevel binary signals has to distinguish between three levels $(+A, -A, 0)$ instead of just two levels in the signaling formats previously discussed. Because of this, the multilevel binary signal requires approximately 3 dB more signal power than a two-valued signal for the same probability of bit error. This is illustrated in Figure 5.4. Put another way, the bit error rate for NRZ codes, at a given signal-to-noise ratio, is significantly less than that for multilevel binary.

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[3]These terms are not used consistently in the literature. In some books, these two terms are used for different encoding schemes than those defined here, and a variety of terms have been used for the two schemes illustrated in Figure 5.2. The nomenclature used here corresponds to the usage in various ITU-T standards documents.

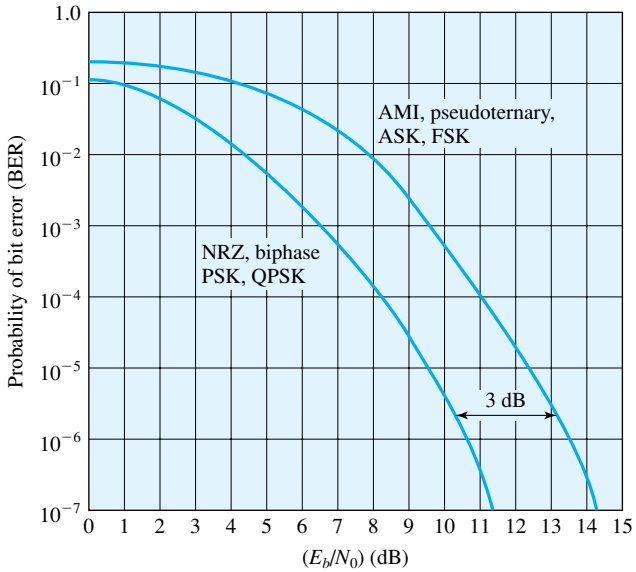


Figure 5.4 Theoretical Bit Error Rate for Various Encoding Schemes

Biphase

There is another set of coding techniques, grouped under the term *biphase*, that overcomes the limitations of NRZ codes. Two of these techniques, Manchester and differential Manchester, are in common use.

In the **Manchester** code, there is a transition at the middle of each bit period. The midbit transition serves as a clocking mechanism and also as data: a low-to-high transition represents a 1, and a high-to-low transition represents a 0.[4] In **differential Manchester**, the midbit transition is used only to provide clocking. The encoding of a 0 is represented by the presence of a transition at the beginning of a bit period, and a 1 is represented by the absence of a transition at the beginning of a bit period. Differential Manchester has the added advantage of employing differential encoding.

All of the biphase techniques require at least one transition per bit time and may have as many as two transitions. Thus, the maximum modulation rate is twice that for NRZ; this means that the bandwidth required is correspondingly greater. On the other hand, the biphase schemes have several advantages:

- **Synchronization:** Because there is a predictable transition during each bit time, the receiver can synchronize on that transition. For this reason, the biphase codes are known as self-clocking codes.
- **No dc component:** Biphase codes have no dc component, yielding the benefits described earlier.

[4]The definition of Manchester presented here is the opposite of that used in a number of respectable textbooks, in which a low-to-high transition represents a binary 0 and a high-to-low transition represents a binary 1. Here, we conform to industry practice and to the definition used in the various LAN standards, such as IEEE 802.3.

- **Error detection:** The absence of an expected transition can be used to detect errors. Noise on the line would have to invert both the signal before and after the expected transition to cause an undetected error.

As can be seen from Figure 5.3, the bandwidth for biphase codes is reasonably narrow and contains no dc component. However, it is wider than the bandwidth for the multilevel binary codes.

Biphase codes are popular techniques for data transmission. The more common Manchester code has been specified for the IEEE 802.3 (Ethernet) standard for baseband coaxial cable and twisted-pair bus LANs. Differential Manchester has been specified for the IEEE 802.5 token ring LAN, using shielded twisted pair.

Modulation Rate

When signal-encoding techniques are used, a distinction needs to be made between data rate (expressed in bits per second) and modulation rate (expressed in baud). The data rate, or bit rate, is $1/T_b$, where T_b = bit duration. The modulation rate is the rate at which signal elements are generated. Consider, for example, Manchester encoding. The minimum size signal element is a pulse of one-half the duration of a bit interval. For a string of all binary zeroes or all binary ones, a continuous stream of such pulses is generated. Hence the maximum modulation rate for Manchester is $2/T_b$. This situation is illustrated in Figure 5.5, which shows the transmission of a stream of binary 1s at a data rate of 1 Mbps using NRZI and Manchester. In general,

$$D = \frac{R}{L} = \frac{R}{\log_2 M} \tag{5.1}$$

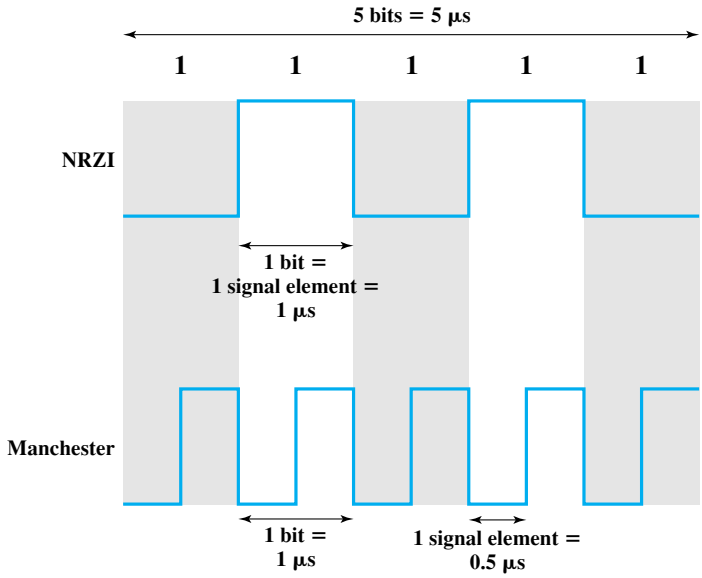


Figure 5.5 A Stream of Binary Ones at 1 Mbps

Table 5.3 Normalized Signal Transition Rate of Various Digital Signal Encoding Schemes

	Minimum	101010 . . .	Maximum
NRZ-L	0 (all 0s or 1s)	1.0	1.0
NRZI	0 (all 0s)	0.5	1.0 (all 1s)
Bipolar-AMI	0 (all 0s)	1.0	1.0
Pseudoternary	0 (all 1s)	1.0	1.0
Manchester	1.0 (1010 . . .)	1.0	2.0 (all 0s or 1s)
Differential Manchester	1.0 (all 1s)	1.5	2.0 (all 0s)

where

$$D = \text{modulation rate, baud}$$
$$R = \text{data rate, bps}$$
$$M = \text{number of different signal elements} = 2^L$$
$$L = \text{number of bits per signal element}$$

One way of characterizing the modulation rate is to determine the average number of transitions that occur per bit time. In general, this will depend on the exact sequence of bits being transmitted. Table 5.3 compares transition rates for various techniques. It indicates the signal transition rate in the case of a data stream of alternating 1s and 0s, and for the data stream that produces the minimum and maximum modulation rate.

Scrambling Techniques

Although the biphase techniques have achieved widespread use in local area network applications at relatively high data rates (up to 10 Mbps), they have not been widely used in long-distance applications. The principal reason for this is that they require a high signaling rate relative to the data rate. This sort of inefficiency is more costly in a long-distance application.

Another approach is to make use of some sort of scrambling scheme. The idea behind this approach is simple: Sequences that would result in a constant voltage level on the line are replaced by filling sequences that will provide sufficient transitions for the receiver's clock to maintain synchronization. The filling sequence must be recognized by the receiver and replaced with the original data sequence. The filling sequence is the same length as the original sequence, so there is no data rate penalty. The design goals for this approach can be summarized as follows:

- No dc component
- No long sequences of zero-level line signals
- No reduction in data rate
- Error-detection capability

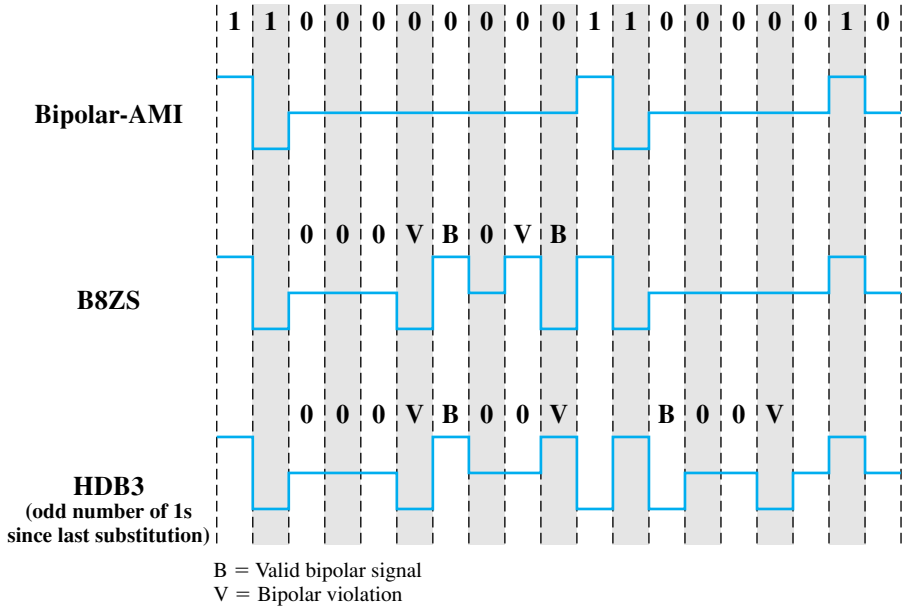


Figure 5.6 Encoding Rules for B8ZS and HDB3

Two techniques are commonly used in long-distance transmission services; these are illustrated in Figure 5.6.

A coding scheme that is commonly used in North America is known as **bipolar with 8-zeros substitution (B8ZS)**. The coding scheme is based on a bipolar-AMI. We have seen that the drawback of the AMI code is that a long string of zeros may result in loss of synchronization. To overcome this problem, the encoding is amended with the following rules:

- If an octet of all zeros occurs and the last voltage pulse preceding this octet was positive, then the eight zeros of the octet are encoded as 000+−0−+.
- If an octet of all zeros occurs and the last voltage pulse preceding this octet was negative, then the eight zeros of the octet are encoded as 000−+0+−.

This technique forces two code violations (signal patterns not allowed in AMI) of the AMI code, an event unlikely to be caused by noise or other transmission impairment. The receiver recognizes the pattern and interprets the octet as consisting of all zeros.

A coding scheme that is commonly used in Europe and Japan is known as the **high-density bipolar-3 zeros (HDB3)** code (Table 5.4). As before, it is based on the use of AMI encoding. In this case, the scheme replaces strings of four zeros with sequences containing one or two pulses. In each case, the fourth zero is replaced with a code violation. In addition, a rule is needed to ensure that successive violations are of alternate polarity so that no dc component is introduced. Thus, if the last violation was positive, this violation must be negative and vice versa. Table 5.4 shows that this condition is tested for by determining (1) whether the number of

Table 5.4 HDB3 Substitution Rules

	Number of Bipolar Pulses (ones) since Last Substitution	
Polarity of Preceding Pulse	**Odd**	**Even**
−	000−	+00+
+	000+	−00−

pulses since the last violation is even or odd and (2) the polarity of the last pulse before the occurrence of the four zeros.

Figure 5.3 shows the spectral properties of these two codes. As can be seen, neither has a dc component. Most of the energy is concentrated in a relatively sharp spectrum around a frequency equal to one-half the data rate. Thus, these codes are well suited to high data rate transmission.

5.2 DIGITAL DATA, ANALOG SIGNALS

We turn now to the case of transmitting digital data using analog signals. The most familiar use of this transformation is for transmitting digital data through the public telephone network. The telephone network was designed to receive, switch, and transmit analog signals in the voice-frequency range of about 300 to 3400 Hz. It is not at present suitable for handling digital signals from the subscriber locations (although this is beginning to change). Thus digital devices are attached to the network via a modem (modulator-demodulator), which converts digital data to analog signals, and vice versa.

For the telephone network, modems are used that produce signals in the voice-frequency range. The same basic techniques are used for modems that produce signals at higher frequencies (e.g., microwave). This section introduces these techniques and provides a brief discussion of the performance characteristics of the alternative approaches.

We mentioned that modulation involves operation on one or more of the three characteristics of a carrier signal: amplitude, frequency, and phase. Accordingly, there are three basic encoding or modulation techniques for transforming digital data into analog signals, as illustrated in Figure 5.7: amplitude shift keying (ASK), frequency shift keying (FSK), and phase shift keying (PSK). In all these cases, the resulting signal occupies a bandwidth centered on the carrier frequency.

Amplitude Shift Keying

In ASK, the two binary values are represented by two different amplitudes of the carrier frequency. Commonly, one of the amplitudes is zero; that is, one binary digit is represented by the presence, at constant amplitude, of the carrier, the other by the absence of the carrier (Figure 5.7a). The resulting transmitted signal for one bit time is

$$\textbf{ASK} \qquad s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary 1} \\ 0 & \text{binary 0} \end{cases} \qquad \textbf{(5.2)}$$

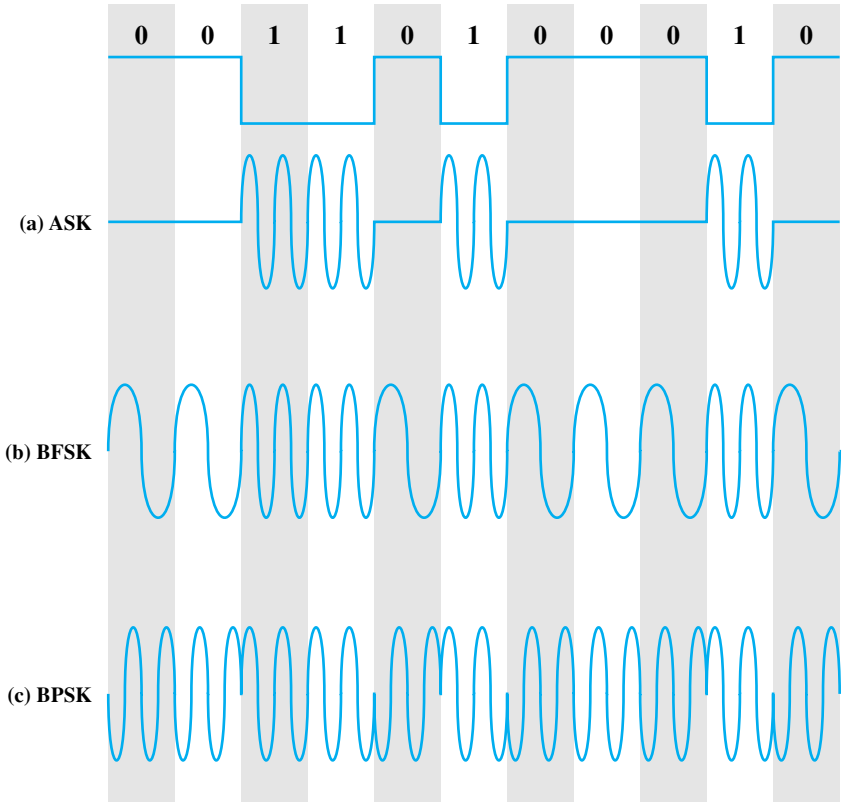


Figure 5.7 Modulation of Analog Signals for Digital Data

where the carrier signal is $A \cos(2\pi f_c t)$. ASK is susceptible to sudden gain changes and is a rather inefficient modulation technique. On voice-grade lines, it is typically used only up to 1200 bps.

The ASK technique is used to transmit digital data over optical fiber. For LED (light-emitting diode) transmitters, Equation (5.2) is valid. That is, one signal element is represented by a light pulse while the other signal element is represented by the absence of light. Laser transmitters normally have a fixed "bias" current that causes the device to emit a low light level. This low level represents one signal element, while a higher-amplitude lightwave represents another signal element.

Frequency Shift Keying

The most common form of FSK is binary FSK (BFSK), in which the two binary values are represented by two different frequencies near the carrier frequency (Figure 5.7b). The resulting transmitted signal for one bit time is

$$\textbf{BFSK} \qquad s(t) = \begin{cases} A \cos(2\pi f_1 t) & \text{binary 1} \\ A \cos(2\pi f_2 t) & \text{binary 0} \end{cases} \qquad \textbf{(5.3)}$$

where f_1 and f_2 are typically offset from the carrier frequency f_c by equal but opposite amounts.

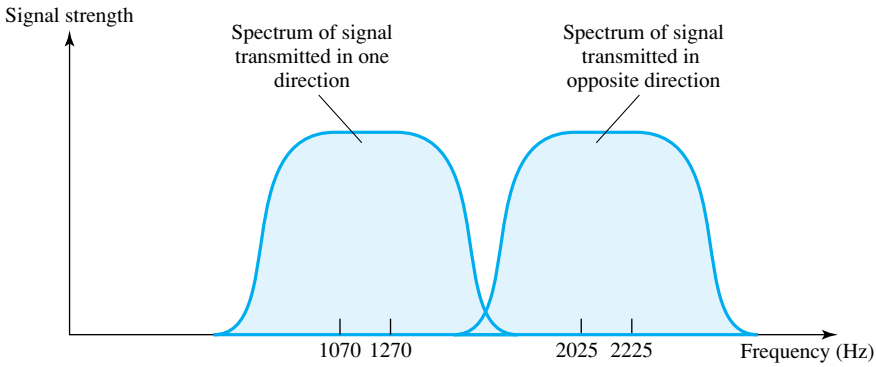


Figure 5.8 Full-Duplex FSK Transmission on a Voice-Grade Line

Figure 5.8 shows an example of the use of BFSK for full-duplex operation over a voice-grade line. The figure is a specification for the Bell System 108 series modems. Recall that a voice-grade line will pass frequencies in the approximate range 300 to 3400 Hz and that *full duplex* means that signals are transmitted in both directions at the same time. To achieve full-duplex transmission, this bandwidth is split. In one direction (transmit or receive), the frequencies used to represent 1 and 0 are centered on 1170 Hz, with a shift of 100 Hz on either side. The effect of alternating between those two frequencies is to produce a signal whose spectrum is indicated as the shaded area on the left in Figure 5.8. Similarly, for the other direction (receive or transmit) the modem uses frequencies shifted 100 Hz to each side of a center frequency of 2125 Hz. This signal is indicated by the shaded area on the right in Figure 5.8. Note that there is little overlap and thus little interference.

BFSK is less susceptible to error than ASK. On voice-grade lines, it is typically used up to 1200 bps. It is also commonly used for high-frequency (3 to 30 MHz) radio transmission. It can also be used at even higher frequencies on local area networks that use coaxial cable.

A signal that is more bandwidth efficient, but also more susceptible to error, is multiple FSK (MFSK), in which more than two frequencies are used. In this case each signaling element represents more than one bit. The transmitted MFSK signal for one signal element time can be defined as follows:

$$\textbf{MFSK} \qquad s_i(t) = A \cos 2\pi f_i t, \qquad 1 \leq i \leq M \tag{5.4}$$

where

$$\begin{aligned} f_i &= f_c + (2i - 1 - M)f_d \\ f_c &= \text{the carrier frequency} \\ f_d &= \text{the difference frequency} \\ M &= \text{number of different signal elements} = 2^L \\ L &= \text{number of bits per signal element} \end{aligned}$$

To match the data rate of the input bit stream, each output signal element is held for a period of $T_s = LT$ seconds, where T is the bit period (data rate $= 1/T$). Thus, one signal element, which is a constant-frequency tone, encodes L bits. The

total bandwidth required is $2Mf_d$. It can be shown that the minimum frequency separation required is $2f_d = 1/T_s$. Therefore, the modulator requires a bandwidth of $W_d = 2Mf_d = M/T_s$.

EXAMPLE 5.1 With $f_c = 250$ kHz, $f_d = 25$ kHz, and $M = 8$ ($L = 3$ bits), we have the following frequency assignments for each of the eight possible 3-bit data combinations:

$f_1 = 75$ kHz	000	$f_2 = 125$ kHz	001
$f_3 = 175$ kHz	010	$f_4 = 225$ kHz	011
$f_5 = 275$ kHz	100	$f_6 = 325$ kHz	101
$f_7 = 375$ kHz	110	$f_8 = 425$ kHz	111

This scheme can support a data rate of $1/T = 2Lf_d = 150$ kbps.

EXAMPLE 5.2 Figure 5.9 shows an example of MFSK with $M = 4$. An input bit stream of 20 bits is encoded 2 bits at a time, with each of the four possible 2-bit combinations transmitted as a different frequency. The display in the figure shows the frequency transmitted (y-axis) as a function of time (x-axis). Each column represents a time unit T_s in which a single 2-bit signal element is transmitted. The shaded rectangle in the column indicates the frequency transmitted during that time unit.

Phase Shift Keying

In PSK, the phase of the carrier signal is shifted to represent data.

Two-Level PSK The simplest scheme uses two phases to represent the two binary digits (Figure 5.7c) and is known as binary phase shift keying. The resulting transmitted signal for one bit time is

$$\textbf{BPSK} \qquad s(t) = \begin{cases} A\cos(2\pi f_c t) \\ A\cos(2\pi f_c t + \pi) \end{cases} = \begin{cases} A\cos(2\pi f_c t) & \text{binary 1} \\ -A\cos(2\pi f_c t) & \text{binary 0} \end{cases} \qquad \textbf{(5.5)}$$

Because a phase shift of 180° (π) is equivalent to flipping the sine wave or multiplying it by -1, the rightmost expressions in Equation (5.5) can be used. This

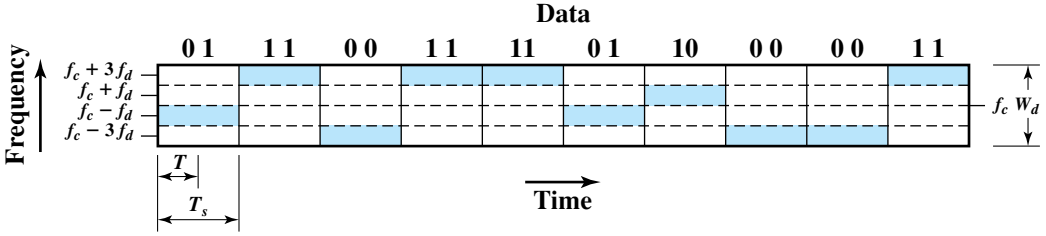


Figure 5.9 MFSK Frequency Use ($M = 4$)

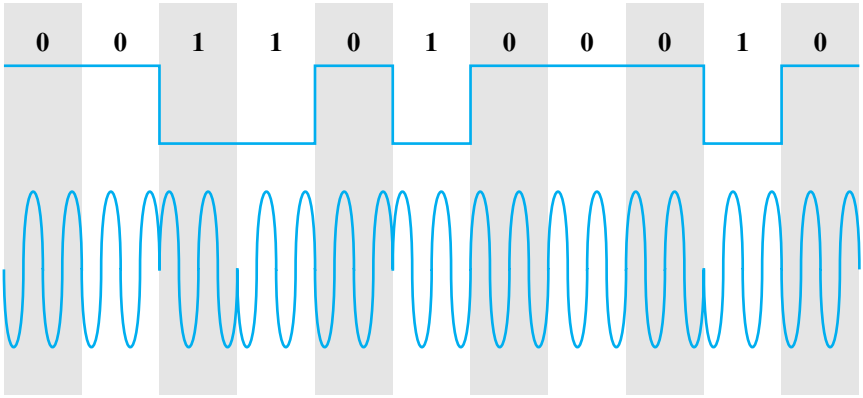


Figure 5.10 Differential Phase-Shift Keying (DPSK)

leads to a convenient formulation. If we have a bit stream, and we define $d(t)$ as the discrete function that takes on the value of +1 for one bit time if the corresponding bit in the bit stream is 1 and the value of −1 for one bit time if the corresponding bit in the bit stream is 0, then we can define the transmitted signal as

$$\textbf{BPSK} \qquad s_d(t) = A\, d(t)\cos(2\pi f_c t) \tag{5.6}$$

An alternative form of two-level PSK is differential PSK (DPSK). Figure 5.10 shows an example. In this scheme, a binary 0 is represented by sending a signal burst of the same phase as the previous signal burst sent. A binary 1 is represented by sending a signal burst of opposite phase to the preceding one. This term *differential* refers to the fact that the phase shift is with reference to the previous bit transmitted rather than to some constant reference signal. In differential encoding, the information to be transmitted is represented in terms of the changes between successive data symbols rather than the signal elements themselves. DPSK avoids the requirement for an accurate local oscillator phase at the receiver that is matched with the transmitter. As long as the preceding phase is received correctly, the phase reference is accurate.

Four-Level PSK More efficient use of bandwidth can be achieved if each signaling element represents more than one bit. For example, instead of a phase shift of 180°, as allowed in BPSK, a common encoding technique, known as quadrature phase shift keying (QPSK), uses phase shifts separated by multiples of $\pi/2$ (90°).

$$\textbf{QPSK} \qquad s(t) = \begin{cases} A\cos\left(2\pi f_c t + \frac{\pi}{4}\right) & 11 \\ A\cos\left(2\pi f_c t + \frac{3\pi}{4}\right) & 01 \\ A\cos\left(2\pi f_c t - \frac{3\pi}{4}\right) & 00 \\ A\cos\left(2\pi f_c t - \frac{\pi}{4}\right) & 10 \end{cases} \tag{5.7}$$

Thus each signal element represents two bits rather than one.

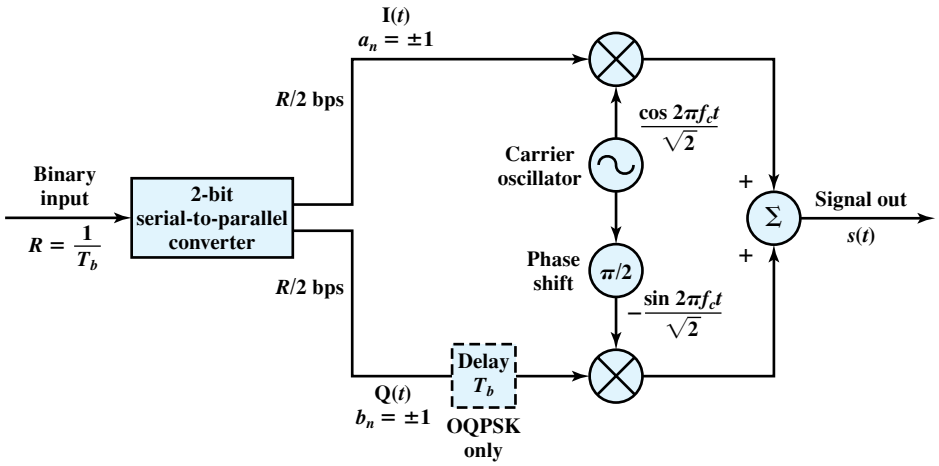


Figure 5.11 QPSK and OQPSK Modulators

Figure 5.11 shows the QPSK modulation scheme in general terms. The input is a stream of binary digits with a data rate of $R = 1/T_b$, where T_b is the width of each bit. This stream is converted into two separate bit streams of $R/2$ bps each, by taking alternate bits for the two streams. The two data streams are referred to as the I (in-phase) and Q (quadrature phase) streams. In the diagram, the upper stream is modulated on a carrier of frequency f_c by multiplying the bit stream by the carrier. For convenience of modulator structure we map binary 1 to $\sqrt{1/2}$ and binary 0 to $-\sqrt{1/2}$. Thus, a binary 1 is represented by a scaled version of the carrier wave and a binary 0 is represented by a scaled version of the negative of the carrier wave, both at a constant amplitude. This same carrier wave is shifted by 90° and used for modulation of the lower binary stream. The two modulated signals are then added together and transmitted. The transmitted signal can be expressed as follows:

$$\textbf{QPSK} \qquad s(t) = \frac{1}{\sqrt{2}} I(t) \cos 2\pi f_c t - \frac{1}{\sqrt{2}} Q(t) \sin 2\pi f_c t$$

Figure 5.12 shows an example of QPSK coding. Each of the two modulated streams is a BPSK signal at half the data rate of the original bit stream. Thus, the combined signals have a symbol rate that is half the input bit rate. Note that from one symbol time to the next, a phase change of as much as 180° (π) is possible.

Figure 5.11 also shows a variation of QPSK known as offset QPSK (OQPSK), or orthogonal QPSK. The difference is that a delay of one bit time is introduced in the Q stream, resulting in the following signal:

$$s(t) = \frac{1}{\sqrt{2}} I(t) \cos 2\pi f_c t - \frac{1}{\sqrt{2}} Q(t - T_b) \sin 2\pi f_c t$$

Because OQPSK differs from QPSK only by the delay in the Q stream, its spectral characteristics and bit error performance are the same as that of QPSK.

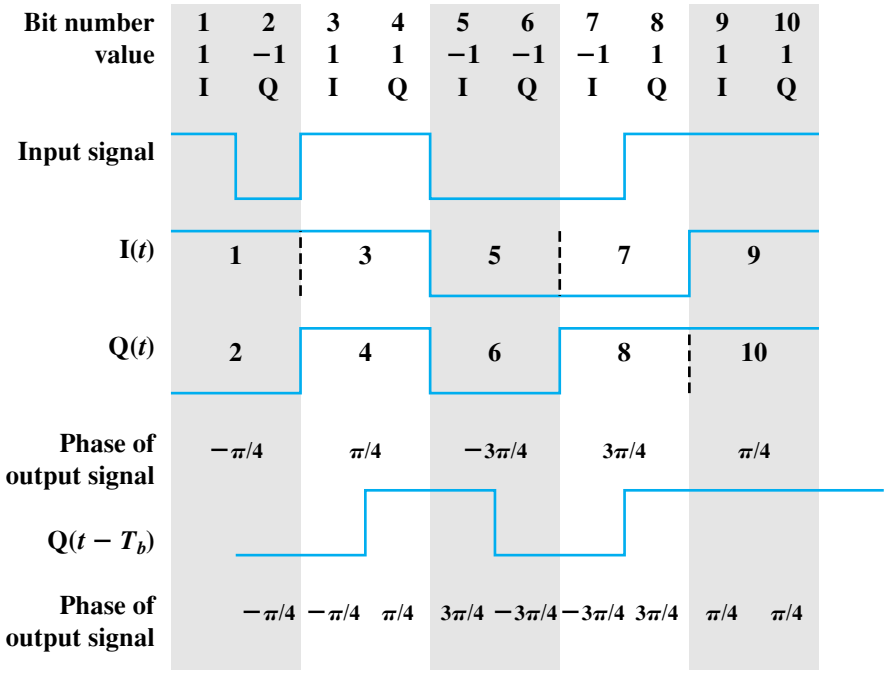


Figure 5.12 Example of QPSK and OQPSK Waveforms

From Figure 5.12, we can observe that only one of two bits in the pair can change sign at any time and thus the phase change in the combined signal never exceeds 90° ($\pi/2$). This can be an advantage because physical limitations on phase modulators make large phase shifts at high transition rates difficult to perform. OQPSK also provides superior performance when the transmission channel (including transmitter and receiver) has significant nonlinear components. The effect of nonlinearities is a spreading of the signal bandwidth, which may result in adjacent channel interference. It is easier to control this spreading if the phase changes are smaller, hence the advantage of OQPSK over QPSK.

Multilevel PSK The use of multiple levels can be extended beyond taking bits two at a time. It is possible to transmit bits three at a time using eight different phase angles. Further, each angle can have more than one amplitude. For example, a standard 9600 bps modem uses 12 phase angles, four of which have two amplitude values, for a total of 16 different signal elements.

This latter example points out very well the difference between the data rate R (in bps) and the modulation rate D (in baud) of a signal. Let us assume that this scheme is being employed with digital input in which each bit is represented by a constant voltage pulse, one level for binary one and one level for binary zero. The data rate is $R = 1/T_b$. However, the encoded signal contains $L = 4$ bits in each signal element using $M = 16$ different combinations of amplitude and phase. The modulation rate can be seen to be $R/4$, because each change of signal element communicates four bits. Thus the line signaling speed is 2400 baud, but the data rate is

9600 bps. This is the reason that higher bit rates can be achieved over voice-grade lines by employing more complex modulation schemes.

Performance

In looking at the performance of various digital-to-analog modulation schemes, the first parameter of interest is the bandwidth of the modulated signal. This depends on a variety of factors, including the definition of bandwidth used and the filtering technique used to create the bandpass signal. We will use some straightforward results from [COUC01].

The transmission bandwidth B_T for ASK is of the form

$$\textbf{ASK} \qquad B_T = (1 + r)R \tag{5.8}$$

where R is the bit rate and r is related to the technique by which the signal is filtered to establish a bandwidth for transmission; typically $0 < r < 1$. Thus the bandwidth is directly related to the bit rate. The preceding formula is also valid for PSK and, under certain assumptions, FSK.

With multilevel PSK (MPSK), significant improvements in bandwidth can be achieved. In general,

$$\textbf{MPSK} \qquad B_T = \left(\frac{1 + r}{L}\right)R = \left(\frac{1 + r}{\log_2 M}\right)R \tag{5.10}$$

where L is the number of bits encoded per signal element and M is the number of different signal elements.

For multilevel FSK (MFSK), we have

$$\textbf{MFSK} \qquad B_T = \left(\frac{(1 + r)M}{\log_2 M}\right)R \tag{5.11}$$

Table 5.5 shows the ratio of data rate, R, to transmission bandwidth for various schemes. This ratio is also referred to as the **bandwidth efficiency**. As the name suggests, this parameter measures the efficiency with which bandwidth can be used to transmit data. The advantage of multilevel signaling methods now becomes clear.

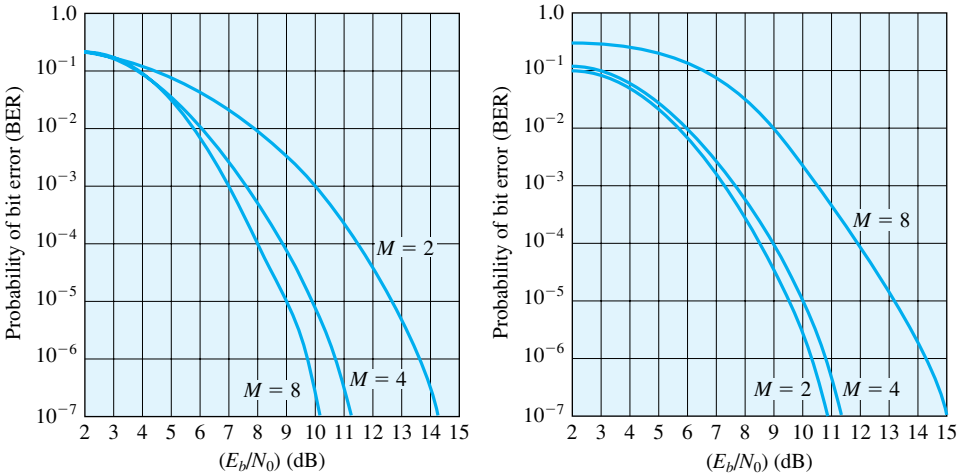
Of course, the preceding discussion refers to the spectrum of the input signal to a communications line. Nothing has yet been said of performance in the presence of noise. Figure 5.4 summarizes some results based on reasonable assumptions concerning the transmission system [COUC01]. Here bit error rate is plotted as a function of the ratio E_b/N_0 defined in Chapter 3. Of course, as that ratio increases, the bit error rate drops. Further, DPSK and BPSK are about 3 dB superior to ASK and BFSK.

Figure 5.13 shows the same information for various levels of M for MFSK and MPSK. There is an important difference. For MFSK, the error probability for a given value E_b/N_0 of decreases as M increases, while the opposite is true for MPSK. On the other hand, comparing Equations (5.10) and (5.11), the bandwidth efficiency of MFSK decreases as M increases, while the opposite is true of MPSK. Thus, in both

Table 5.5 Bandwidth Efficiency (R/B_T) for Various Digital-to-Analog Encoding Schemes

	$r = 0$	$r = 0.5$	$r = 1$
ASK	1.0	0.67	0.5
FSK	0.5	0.33	0.25
Multilevel FSK			
$M = 4, L = 2$	0.5	0.33	0.25
$M = 8, L = 3$	0.375	0.25	0.1875
$M = 16, L = 4$	0.25	0.167	0.125
$M = 32, L = 5$	0.156	0.104	0.078
PSK	1.0	0.67	0.5
Multilevel PSK			
$M = 4, L = 2$	2.00	1.33	1.00
$M = 8, L = 3$	3.00	2.00	1.50
$M = 16, L = 4$	4.00	2.67	2.00
$M = 32, L = 5$	5.00	3.33	2.50

cases, there is a tradeoff between bandwidth efficiency and error performance: An increase in bandwidth efficiency results in an increase in error probability. The fact that these tradeoffs move in opposite directions with respect to the number of levels M for MFSK and MPSK can be derived from the underlying equations. A discussion of the reasons for this difference is beyond the scope of this book. See [SKLA01] for a full treatment.



(a) Multilevel FSK (MFSK) **(b) Multilevel PSK (MPSK)**

Figure 5.13 Theoretical Bit Error Rate for Multilevel FSK and PSK

EXAMPLE 5.3 What is the bandwidth efficiency for FSK, ASK, PSK, and QPSK for a bit error rate of 10^{-7} on a channel with an SNR of 12 dB?

Using Equation (3.2), we have

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} = 12 \text{ dB} - \left(\frac{R}{B_T}\right)_{\text{dB}}$$

For FSK and ASK, from Figure 5.4,

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} = 14.2 \text{ dB}$$

$$\left(\frac{R}{B_T}\right)_{\text{dB}} = -2.2 \text{ dB}$$

$$\frac{R}{B_T} = 0.6$$

For PSK, from Figure 5.4,

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} = 11.2 \text{ dB}$$

$$\left(\frac{R}{B_T}\right)_{\text{dB}} = 0.8 \text{ dB}$$

$$\frac{R}{B_T} = 1.2$$

The result for QPSK must take into account that the baud rate $D = R/2$. Thus

$$\frac{R}{B_T} = 2.4$$

As the preceding example shows, ASK and FSK exhibit the same bandwidth efficiency, PSK is better, and even greater improvement can be achieved with multilevel signaling.

It is worthwhile to compare these bandwidth requirements with those for digital signaling. A good approximation is

$$B_T = 0.5(1 + r)D$$

where D is the modulation rate. For NRZ, $D = R$, and we have

$$\frac{R}{B_T} = \frac{2}{1 + r}$$

Thus digital signaling is in the same ballpark, in terms of bandwidth efficiency, as ASK, FSK, and PSK. A significant advantage for analog signaling is seen with multilevel techniques.

Quadrature Amplitude Modulation

Quadrature amplitude modulation (QAM) is a popular analog signaling technique that is used in the asymmetric digital subscriber line (ADSL), described in Chapter 8, and in some wireless standards. This modulation technique is a combination of ASK and PSK. QAM can also be considered a logical extension of QPSK. QAM takes advantage of the fact that it is possible to send two different signals simultaneously on the same carrier frequency, by using two copies of the carrier frequency, one shifted by 90° with respect to the other. For QAM, each carrier is ASK modulated. The two independent signals are simultaneously transmitted over the same medium. At the receiver, the two signals are demodulated and the results combined to produce the original binary input.

Figure 5.14 shows the QAM modulation scheme in general terms. The input is a stream of binary digits arriving at a rate of R bps. This stream is converted into two separate bit streams of $R/2$ bps each, by taking alternate bits for the two streams. In the diagram, the upper stream is ASK modulated on a carrier of frequency f_c by multiplying the bit stream by the carrier. Thus, a binary zero is represented by the absence of the carrier wave and a binary one is represented by the presence of the carrier wave at a constant amplitude. This same carrier wave is shifted by 90° and used for ASK modulation of the lower binary stream. The two modulated signals are then added together and transmitted. The transmitted signal can be expressed as follows:

$$\textbf{QAM} \qquad s(t) = d_1(t)\cos 2\pi f_c t + d_2(t)\sin 2\pi f_c t$$

If two-level ASK is used, then each of the two streams can be in one of two states and the combined stream can be in one of $4 = 2 \times 2$ states. This is essentially QPSK. If four-level ASK is used (i.e., four different amplitude levels), then the combined stream can be in one of $16 = 4 \times 4$ states. Systems using 64 and even 256 states have been implemented. The greater the number of states, the higher the data rate that is possible within a given bandwidth. Of course, as discussed previously, the greater the number of states, the higher the potential error rate due to noise and attenuation.

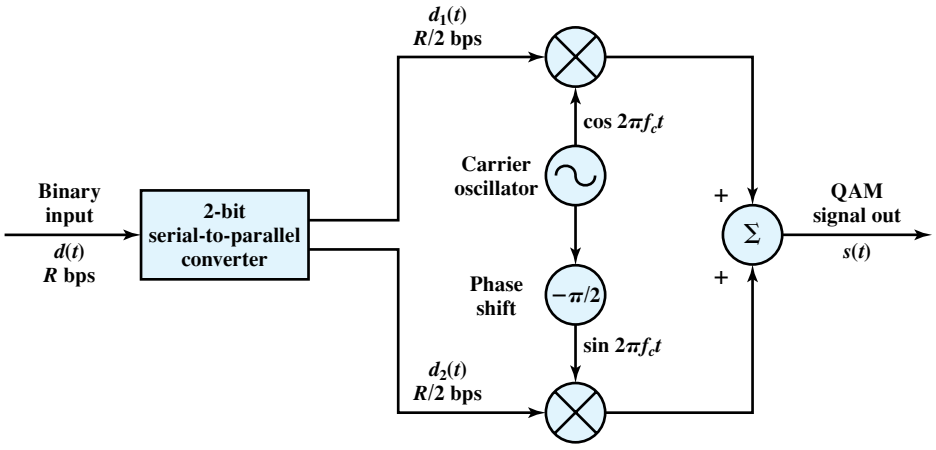


Figure 5.14 QAM Modulator

5.3 ANALOG DATA, DIGITAL SIGNALS

In this section we examine the process of transforming analog data into digital signals. Strictly speaking, it might be more correct to refer to this as a process of converting analog data into digital data; this process is known as digitization. Once analog data have been converted into digital data, a number of things can happen. The three most common are as follows:

1. The digital data can be transmitted using NRZ-L. In this case, we have in fact gone directly from analog data to a digital signal.
2. The digital data can be encoded as a digital signal using a code other than NRZ-L. Thus an extra step is required.
3. The digital data can be converted into an analog signal, using one of the modulation techniques discussed in Section 5.2.

This last, seemingly curious, procedure is illustrated in Figure 5.15, which shows voice data that are digitized and then converted to an analog ASK signal. This allows digital transmission in the sense defined in Chapter 3. The voice data, because they have been digitized, can be treated as digital data, even though transmission requirements (e.g., use of microwave) dictate that an analog signal be used.

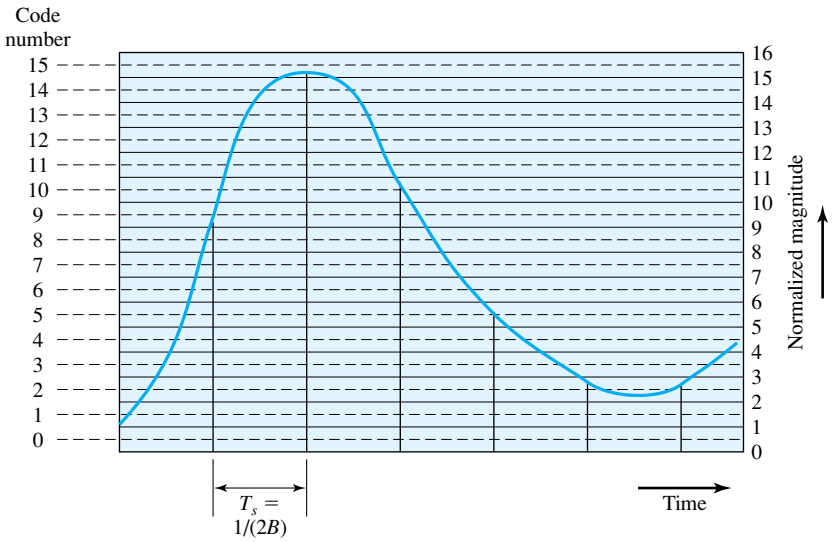
The device used for converting analog data into digital form for transmission, and subsequently recovering the original analog data from the digital, is known as a **codec** (coder-decoder). In this section we examine the two principal techniques used in codecs, pulse code modulation and delta modulation. The section closes with a discussion of comparative performance.

Pulse Code Modulation

Pulse code modulation (PCM) is based on the sampling theorem:

SAMPLING THEOREM: If a signal $f(t)$ is sampled at regular intervals of time and at a rate higher than twice the highest signal frequency, then the samples contain all the information of the original signal. The function $f(t)$ may be reconstructed from these samples by the use of a lowpass filter.

Figure 5.15 Digitizing Analog Data



PAM value	1.1	9.2	15.2	10.8	5.6	2.8	2.7
Quantized code number	1	9	15	10	5	2	2
PCM code	0001	1001	1111	1010	0101	0010	0010

Figure 5.16 Pulse Code Modulation Example

For the interested reader, a proof is provided in Appendix F. If voice data are limited to frequencies below 4000 Hz, a conservative procedure for intelligibility, 8000 samples per second would be sufficient to characterize the voice signal completely. Note, however, that these are analog samples, called **pulse amplitude modulation (PAM)** samples. To convert to digital, each of these analog samples must be assigned a binary code.

Figure 5.16 shows an example in which the original signal is assumed to be bandlimited with a bandwidth of B. PAM samples are taken at a rate of $2B$, or once every $T_s = 1/2B$ seconds. Each PAM sample is approximated by being *quantized* into one of 16 different levels. Each sample can then be represented by 4 bits. But because the quantized values are only approximations, it is impossible to recover the original signal exactly. By using an 8-bit sample, which allows 256 quantizing levels, the quality of the recovered voice signal is comparable with that achieved via analog transmission. Note that this implies that a data rate of 8000 samples per second $\times$ 8 bits per sample = 64 kbps is needed for a single voice signal.

Thus, PCM starts with a continuous-time, continuous-amplitude (analog) signal, from which a digital signal is produced (Figure 5.17). The digital signal consists of blocks of n bits, where each n-bit number is the amplitude of a PCM pulse. On reception, the process is reversed to reproduce the analog signal. Notice, however, that this process violates the terms of the sampling theorem. By quantizing the PAM pulse, the original signal is now only approximated and cannot be recovered exactly. This effect is known as **quantizing error** or **quantizing**

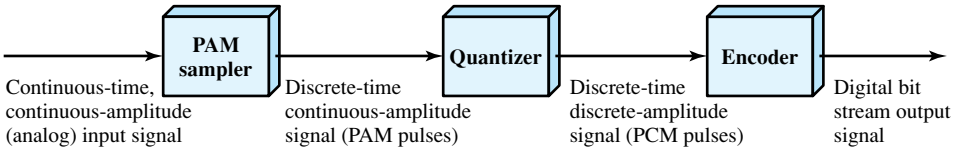


Figure 5.17 PCM Block Diagram

noise. The signal-to-noise ratio for quantizing noise can be expressed as [GIBS93]

$$\text{SNR}_{\text{dB}} = 20 \log 2^n + 1.76 \text{ dB} = 6.02n + 1.76 \text{ dB}$$

Thus each additional bit used for quantizing increases SNR by about 6 dB, which is a factor of 4.

Typically, the PCM scheme is refined using a technique known as nonlinear encoding, which means, in effect, that the quantization levels are not equally spaced. The problem with equal spacing is that the mean absolute error for each sample is the same, regardless of signal level. Consequently, lower amplitude values are relatively more distorted. By using a greater number of quantizing steps for signals of low amplitude, and a smaller number of quantizing steps for signals of large amplitude, a marked reduction in overall signal distortion is achieved (e.g., see Figure 5.18).

The same effect can be achieved by using uniform quantizing but companding (compressing-expanding) the input analog signal. Companding is a process that compresses the intensity range of a signal by imparting more gain to weak signals than to strong signals on input. At output, the reverse operation is performed. Figure 5.19 shows typical companding functions. Note that the effect on the input side is to compress the sample so that the higher values are reduced with respect

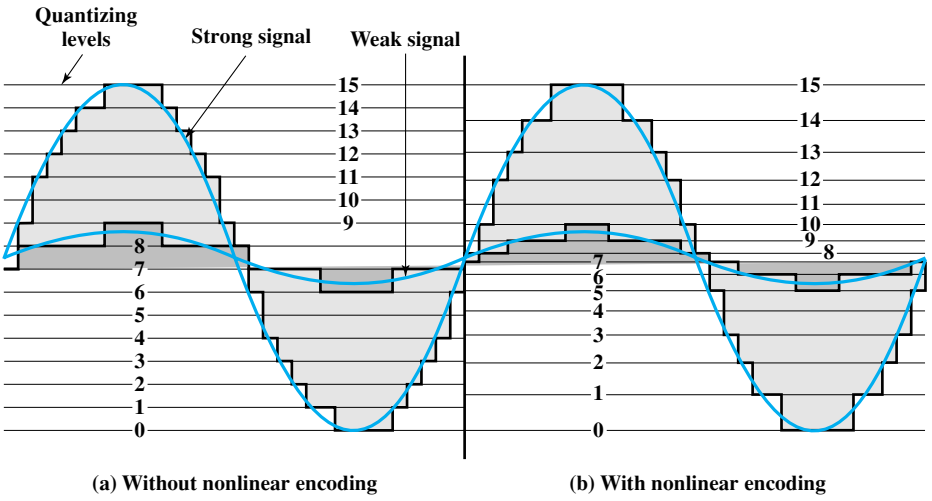


Figure 5.18 Effect of Nonlinear Coding

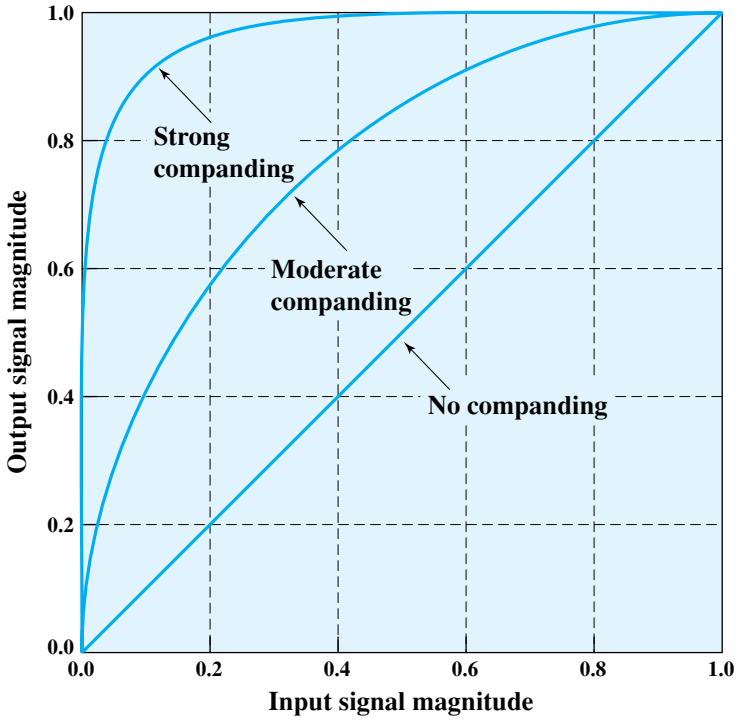


Figure 5.19 Typical Companding Functions

to the lower values. Thus, with a fixed number of quantizing levels, more levels are available for lower-level signals. On the output side, the compander expands the samples so the compressed values are restored to their original values.

Nonlinear encoding can significantly improve the PCM SNR ratio. For voice signals, improvements of 24 to 30 dB have been achieved.

Delta Modulation (DM)

A variety of techniques have been used to improve the performance of PCM or to reduce its complexity. One of the most popular alternatives to PCM is delta modulation (DM).

With delta modulation, an analog input is approximated by a staircase function that moves up or down by one quantization level (δ) at each sampling interval (T_s). An example is shown in Figure 5.20, where the staircase function is overlaid on the original analog waveform. The important characteristic of this staircase function is that its behavior is binary: At each sampling time, the function moves up or down a constant amount δ. Thus, the output of the delta modulation process can be represented as a single binary digit for each sample. In essence, a bit stream is produced by approximating the derivative of an analog signal rather than its amplitude: A 1 is generated if the staircase function is to go up during the next interval; a 0 is generated otherwise.

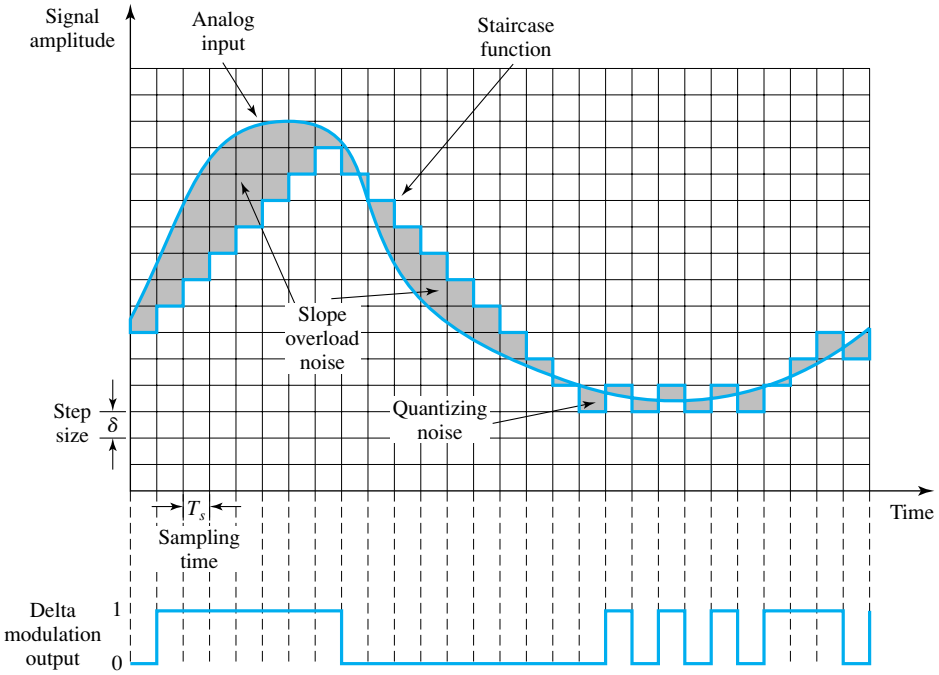


Figure 5.20 Example of Delta Modulation

The transition (up or down) that occurs at each sampling interval is chosen so that the staircase function tracks the original analog waveform as closely as possible. Figure 5.21 illustrates the logic of the process, which is essentially a feedback mechanism. For transmission, the following occurs: At each sampling time, the analog input is compared to the most recent value of the approximating staircase function. If the value of the sampled waveform exceeds that of the staircase function, a 1 is generated; otherwise, a 0 is generated. Thus, the staircase is always changed in the direction of the input signal. The output of the DM process is therefore a binary sequence that can be used at the receiver to reconstruct the staircase function. The staircase function can then be smoothed by some type of integration process or by passing it through a lowpass filter to produce an analog approximation of the analog input signal.

There are two important parameters in a DM scheme: the size of the step assigned to each binary digit, δ, and the sampling rate. As Figure 5.20 illustrates, δ must be chosen to produce a balance between two types of errors or noise. When the analog waveform is changing very slowly, there will be quantizing noise. This noise increases as δ is increased. On the other hand, when the analog waveform is changing more rapidly than the staircase can follow, there is slope overload noise. This noise increases as δ is decreased.

It should be clear that the accuracy of the scheme can be improved by increasing the sampling rate. However, this increases the data rate of the output signal.

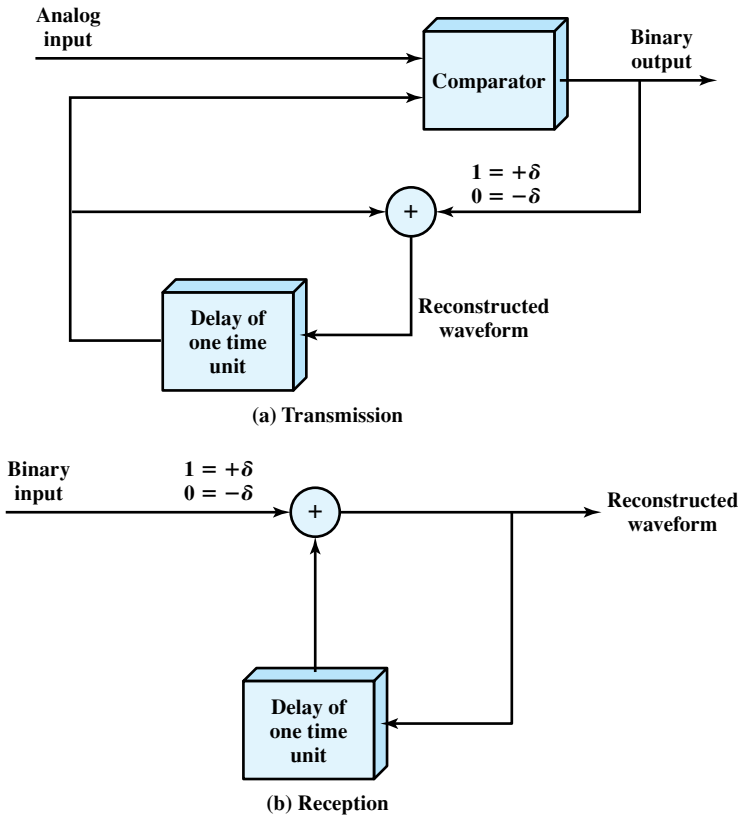


Figure 5.21 Delta Modulation

The principal advantage of DM over PCM is the simplicity of its implementation. In general, PCM exhibits better SNR characteristics at the same data rate.

Performance

Good voice reproduction via PCM can be achieved with 128 quantization levels, or 7-bit coding ($2^7 = 128$). A voice signal, conservatively, occupies a bandwidth of 4 kHz. Thus, according to the sampling theorem, samples should be taken at a rate of 8000 samples per second. This implies a data rate of $8000 \times 7 = 56$ kbps for the PCM-encoded digital data.

Consider what this means from the point of view of bandwidth requirement. An analog voice signal occupies 4 kHz. Using PCM this 4-kHz analog signal can be converted into a 56-kbps digital signal. But using the Nyquist criterion from Chapter 3, this digital signal could require on the order of 28 kHz of bandwidth. Even more severe differences are seen with higher bandwidth signals. For example, a common PCM scheme for color television uses 10-bit codes, which works out to 92 Mbps for a 4.6-MHz bandwidth signal. In spite of these numbers, digital techniques continue to grow in popularity for transmitting analog data. The principal reasons for this are as follows:

- Because repeaters are used instead of amplifiers, there is no cumulative noise.
- As we shall see, time division multiplexing (TDM) is used for digital signals instead of the frequency division multiplexing (FDM) used for analog signals. With TDM, there is no intermodulation noise, whereas we have seen that this is a concern for FDM.
- The conversion to digital signaling allows the use of the more efficient digital switching techniques.

Furthermore, techniques have been developed to provide more efficient codes. In the case of voice, a reasonable goal appears to be in the neighborhood of 4 kbps. With video, advantage can be taken of the fact that from frame to frame, most picture elements will not change. Interframe coding techniques should allow the video requirement to be reduced to about 15 Mbps, and for slowly changing scenes, such as found in a video teleconference, down to 64 kbps or less.

As a final point, we mention that in many instances, the use of a telecommunications system will result in both digital-to-analog and analog-to-digital processing. The overwhelming majority of local terminations into the telecommunications network is analog, and the network itself uses a mixture of analog and digital techniques. Thus digital data at a user's terminal may be converted to analog by a modem, subsequently digitized by a codec, and perhaps suffer repeated conversions before reaching its destination.

Thus, telecommunication facilities handle analog signals that represent both voice and digital data. The characteristics of the waveforms are quite different. Whereas voice signals tend to be skewed to the lower portion of the bandwidth (Figure 3.9), analog encoding of digital signals has a more uniform spectral content over the bandwidth and therefore contains more high-frequency components. Studies have shown that, because of the presence of these higher frequencies, PCM-related techniques are preferable to DM-related techniques for digitizing analog signals that represent digital data.

5.4 ANALOG DATA, ANALOG SIGNALS

Modulation has been defined as the process of combining an input signal $m(t)$ and a carrier at frequency f_c to produce a signal $s(t)$ whose bandwidth is (usually) centered on f_c. For digital data, the motivation for modulation should be clear: When only analog transmission facilities are available, modulation is required to convert the digital data to analog form. The motivation when the data are already analog is less clear. After all, voice signals are transmitted over telephone lines at their original spectrum (referred to as baseband transmission). There are two principal reasons for analog modulation of analog signals:

- A higher frequency may be needed for effective transmission. For unguided transmission, it is virtually impossible to transmit baseband signals; the required antennas would be many kilometers in diameter.
- Modulation permits frequency division multiplexing, an important technique explored in Chapter 8.

In this section we look at the principal techniques for modulation using analog data: amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM). As before, the three basic characteristics of a signal are used for modulation.

Amplitude Modulation

Amplitude modulation (AM) is the simplest form of modulation and is depicted in Figure 5.22. Mathematically, the process can be expressed as

$$\textbf{AM} \qquad s(t) = [1 + n_a x(t)]\cos 2\pi f_c t \tag{5.12}$$

where $\cos 2\pi f_c t$ is the carrier and $x(t)$ is the input signal (carrying data), both normalized to unity amplitude. The parameter n_a, known as the **modulation index**, is the ratio of the amplitude of the input signal to the carrier. Corresponding to our previous notation, the input signal is $m(t) = n_a x(t)$. The "1" in the Equation (5.12) is a dc component that prevents loss of information, as explained subsequently. This scheme is also known as double sideband transmitted carrier (DSBTC).

EXAMPLE 5.4 Derive an expression for $s(t)$ if $x(t)$ is the amplitude-modulating signal $\cos 2\pi f_m t$. We have

$$s(t) = [1 + n_a \cos 2\pi f_m t]\cos 2\pi f_c t$$

By trigonometric identity, this may be expanded to

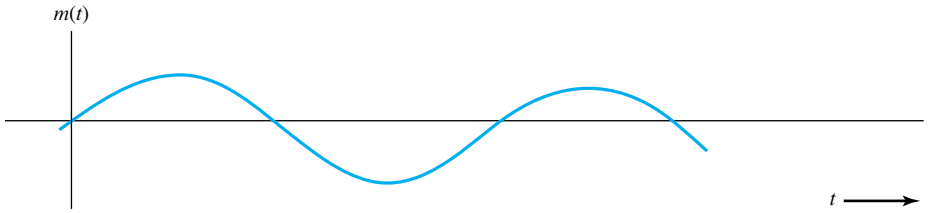
$$s(t) = \cos 2\pi f_c t + \frac{n_a}{2}\cos 2\pi (f_c - f_m)t + \frac{n_a}{2}\cos 2\pi (f_c + f_m)t$$

The resulting signal has a component at the original carrier frequency plus a pair of components each spaced f_m hertz from the carrier.

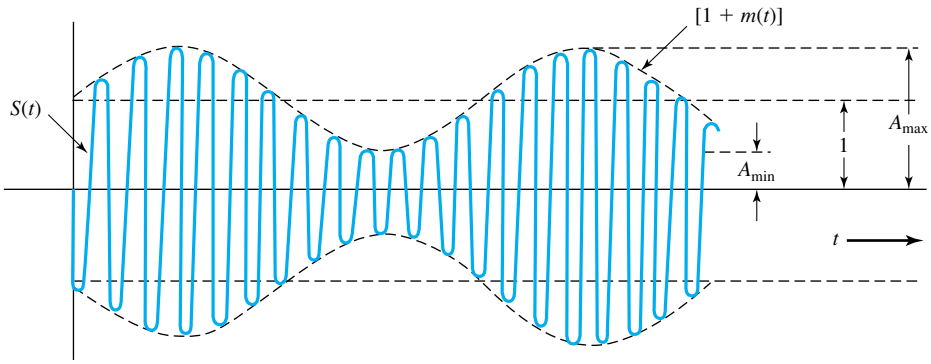
From Equation (5.12) and Figure 5.22, it can be seen that AM involves the multiplication of the input signal by the carrier. The envelope of the resulting signal is $[1 + n_a x(t)]$ and, as long as $n_a < 1$, the envelope is an exact reproduction of the original signal. If $n_a > 1$, the envelope will cross the time axis and information is lost.

It is instructive to look at the spectrum of the AM signal. An example is shown in Figure 5.23. The spectrum consists of the original carrier plus the spectrum of the input signal translated to f_c. The portion of the spectrum for $|f| > |f_c|$ is the *upper sideband,* and the portion of the spectrum for $|f| < |f_c|$ is *lower sideband.* Both the upper and lower sidebands are replicas of the original spectrum $M(f)$, with the lower sideband being frequency reversed. As an example, consider a voice signal with a bandwidth that extends from 300 to 3000 Hz being modulated on a 60-kHz carrier. The resulting signal contains an upper sideband of 60.3 to 63 kHz, a lower sideband of 57 to 59.7 kHz, and the 60-kHz carrier. An important relationship is

$$P_t = P_c\left(1 + \frac{n_a^2}{2}\right)$$

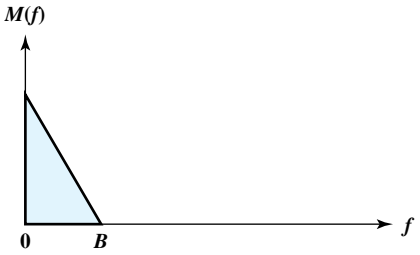


(a) Sinusoidal modulating wave

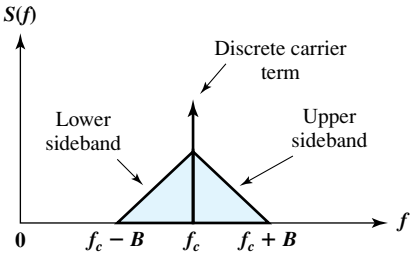


(b) Resulting AM signal

Figure 5.22 Amplitude Modulation



(a) Spectrum of modulating signal



(b) Spectrum of AM signal with carrier at f_c

Figure 5.23 Spectrum of an AM Signal

where P_t is the total transmitted power in $s(t)$ and P_c is the transmitted power in the carrier. We would like n_a as large as possible so that most of the signal power is used to carry information. However, n_a must remain below 1.

It should be clear that $s(t)$ contains unnecessary components, because each of the sidebands contains the complete spectrum of $m(t)$. A popular variant of AM, known as single sideband (SSB), takes advantage of this fact by sending only one of the sidebands, eliminating the other sideband and the carrier. The principal advantages of this approach are as follows:

- Only half the bandwidth is required, that is, $B_T = B$, where B is the bandwidth of the original signal. For DSBTC, $B_T = 2B$.
- Less power is required because no power is used to transmit the carrier or the other sideband. Another variant is double sideband suppressed carrier (DSBSC), which filters out the carrier frequency and sends both sidebands. This saves some power but uses as much bandwidth as DSBTC.

The disadvantage of suppressing the carrier is that the carrier can be used for synchronization purposes. For example, suppose that the original analog signal is an ASK waveform encoding digital data. The receiver needs to know the starting point of each bit time to interpret the data correctly. A constant carrier provides a clocking mechanism by which to time the arrival of bits. A compromise approach is vestigial sideband (VSB), which uses one sideband and a reduced-power carrier.

Angle Modulation

Frequency modulation (FM) and phase modulation (PM) are special cases of angle modulation. The modulated signal is expressed as

$$\textbf{Angle Modulation} \qquad s(t) = A_c \cos[2\pi f_c t + \phi(t)] \tag{5.13}$$

For phase modulation, the phase is proportional to the modulating signal:

$$\textbf{PM} \qquad \phi(t) = n_p m(t) \tag{5.14}$$

where n_p is the phase modulation index.

For frequency modulation, the derivative of the phase is proportional to the modulating signal:

$$\textbf{FM} \qquad \phi'(t) = n_f m(t) \tag{5.15}$$

where n_f is the frequency modulation index and $\phi'(t)$ is the derivative of $\phi(t)$.

For those who wish a more detailed mathematical explanation of the preceding, consider the following. The phase of $s(t)$ at any instant is just $2\pi f_c t + \phi(t)$. The instantaneous phase deviation from the carrier signal is $\phi(t)$. In PM, this instantaneous phase deviation is proportional to $m(t)$. Because frequency can be defined as the rate of change of phase of a signal, the instantaneous frequency of $s(t)$ is

$$2\pi f_i(t) = \frac{d}{dt}[2\pi f_c t + \phi(t)]$$

$$f_i(t) = f_c + \frac{1}{2\pi}\phi'(t)$$

and the instantaneous frequency deviation from the carrier frequency is $\phi'(t)$, which in FM is proportional to $m(t)$.

Figure 5.24 illustrates amplitude, phase, and frequency modulation by a sine wave. The shapes of the FM and PM signals are very similar. Indeed, it is impossible to tell them apart without knowledge of the modulation function.

Several observations about the FM process are in order. The peak deviation ΔF can be seen to be

$$\Delta F = \frac{1}{2\pi} n_f A_m \text{ Hz}$$

where A_m is the maximum value of $m(t)$. Thus an increase in the magnitude of $m(t)$ will increase ΔF, which, intuitively, should increase the transmitted bandwidth B_T. However, as should be apparent from Figure 5.24, this will not increase the average power level of the FM signal, which is $A_c^2/2$. This is distinctly different from AM, where the level of modulation affects the power in the AM signal but does not affect its bandwidth.

EXAMPLE 5.5 Derive an expression for $s(t)$ if $\phi(t)$ is the phase-modulating signal $n_p \cos 2\pi f_m t$. Assume that $A_c = 1$. This can be seen directly to be

$$s(t) = \cos[2\pi f_c t + n_p \cos 2\pi f_m t]$$

The instantaneous phase deviation from the carrier signal is $n_p \cos 2\pi f_m t$. The phase angle of the signal varies from its unmodulated value in a simple sinusoidal fashion, with the peak phase deviation equal to n_p.

The preceding expression can be expanded using Bessel's trigonometric identities:

$$s(t) = \sum_{n=-\infty}^{\infty} J_n(n_p) \cos\left(2\pi f_c t + 2\pi n f_m t + \frac{n\pi}{2}\right)$$

where $J_n(n_p)$ is the nth-order Bessel function of the first kind. Using the property

$$J_{-n}(x) = (-1)^n J_n(x)$$

this can be rewritten as

$$s(t) = J_0(n_p) \cos 2\pi f_c t + \sum_{n=1}^{\infty} J_n(n_p) \left[\cos\left(2\pi(f_c + nf_m)t + \frac{n\pi}{2}\right) + \cos\left(2\pi(f_c - nf_m)t + \frac{(n+2)\pi}{2}\right)\right]$$

The resulting signal has a component at the original carrier frequency plus a set of sidebands displaced from f_c by all possible multiples of f_m. For $n_p \ll 1$, the higher-order terms fall off rapidly.

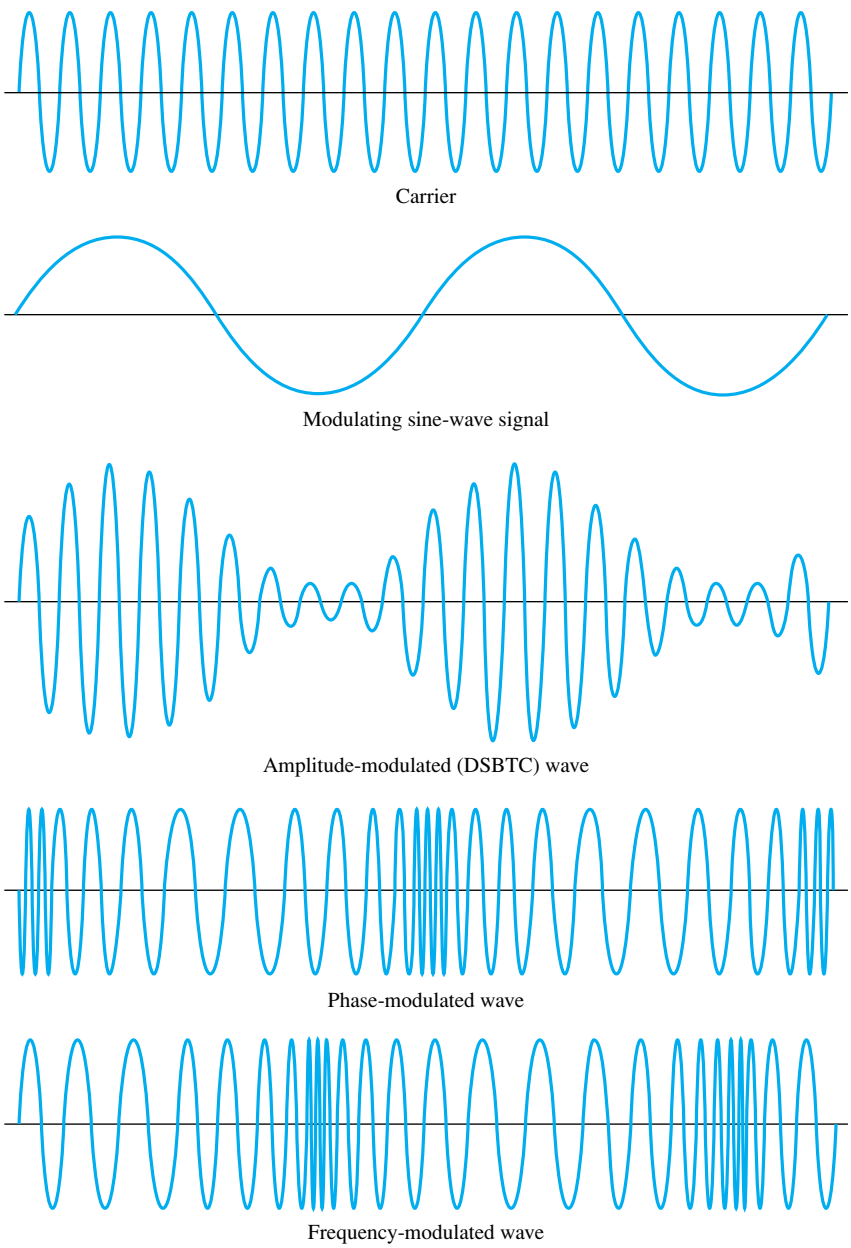


Figure 5.24 Amplitude, Phase, and Frequency Modulation of a Sine-Wave Carrier by a Sine-Wave Signal

EXAMPLE 5.6 Derive an expression for $s(t)$ if $\phi'(t)$ is the frequency modulating signal $-n_f \sin 2\pi f_m t$. The form of $\phi'(t)$ was chosen for convenience. We have

$$\phi(t) = -\int n_f \sin 2\pi f_m t \, dt = \frac{n_f}{2\pi f_m} \cos 2\pi f_m t$$

Thus

$$s(t) = \cos\left[2\pi f_c t + \frac{n_f}{2\pi f_m} \cos 2\pi f_m t\right]$$
$$= \cos\left[2\pi f_c t + \frac{\Delta F}{f_m} \cos 2\pi f_m t\right]$$

The instantaneous frequency deviation from the carrier signal is $-n_f \sin 2\pi f_m t$. The frequency of the signal varies from its unmodulated value in a simple sinusoidal fashion, with the peak frequency deviation equal to n_f radians/second.

The equation for the FM signal has the identical form as for the PM signal, with $\Delta F/f_m$ substituted for n_p. Thus the Bessel expansion is the same.

As with AM, both FM and PM result in a signal whose bandwidth is centered at f_c. However, we can now see that the magnitude of that bandwidth is very different. Amplitude modulation is a linear process and produces frequencies that are the sum and difference of the carrier signal and the components of the modulating signal. Hence, for AM,

$$B_T = 2B$$

However, angle modulation includes a term of the form $\cos(\phi(t))$, which is nonlinear and will produce a wide range of frequencies. In essence, for a modulating sinusoid of frequency f_m, $s(t)$ will contain components at $f_c + f_m$, $f_c + 2f_m$, and so on. In the most general case, infinite bandwidth is required to transmit an FM or PM signal. As a practical matter, a very good rule of thumb, known as Carson's rule [COUC01], is

$$B_T = 2(\beta + 1)B$$

where

$$\beta = \begin{cases} n_p A_m & \text{for PM} \\ \dfrac{\Delta F}{B} = \dfrac{n_f A_m}{2\pi B} & \text{for FM} \end{cases}$$

We can rewrite the formula for FM as

$$B_T = 2\Delta F + 2B \qquad \textbf{(5.16)}$$

Thus both FM and PM require greater bandwidth than AM.

5.5 RECOMMENDED READING

It is difficult, for some reason, to find solid treatments of digital-to-digital encoding schemes. Useful accounts include [SKLA01] and [BERG96].

There are many good references on analog modulation schemes for digital data. Good choices are [COUC01], [XION00], and [PROA05]; these three also provide comprehensive treatment of digital and analog modulation schemes for analog data.

An instructive treatment of the concepts of bit rate, baud, and bandwidth is [FREE98]. A recommended tutorial that expands on the concepts treated in the past few chapters relating to bandwidth efficiency and encoding schemes is [SKLA93].

BERG96 Bergmans, J. *Digital Baseband Transmission and Recording.* Boston: Kluwer, 1996.

COUC01 Couch, L. *Digital and Analog Communication Systems.* Upper Saddle River, NJ: Prentice Hall, 2001.

FREE98 Freeman, R. "Bits, Symbols, Baud, and Bandwidth." *IEEE Communications Magazine*, April 1998.

PROA05 Proakis, J. *Fundamentals of Communication Systems.* Upper Saddle River, NJ: Prentice Hall, 2005.

SKLA93 Sklar, B. "Defining, Designing, and Evaluating Digital Communication Systems." *IEEE Communications Magazine*, November 1993.

SKLA01 Sklar, B. *Digital Communications: Fundamentals and Applications.* Englewood Cliffs, NJ: Prentice Hall, 2001.

XION00 Xiong, F. *Digital Modulation Techniques.* Boston: Artech House, 2000.

5.6 KEY TERMS, REVIEW QUESTIONS, AND PROBLEMS

Key Terms

alternate mark inversion (AMI)
amplitude modulation (AM)
amplitude shift keying (ASK)
angle modulation
bandwidth efficiency
baseband signal
biphase
bipolar-AMI
bipolar with 8-zeros substitution (B8ZS)
bit error rate (BER)
carrier frequency
delta modulation (DM)
differential encoding
differential Manchester
differential PSK (DPSK)
frequency modulation (FM)
frequency shift keying (FSK)
high-density bipolar-3 zeros (HDB3)
Manchester
modulation
modulation rate
multilevel binary
nonreturn to zero (NRZ)
nonreturn to zero, inverted (NRZI)
nonreturn to zero-level (NRZ-L)
phase modulation (PM)
phase shift keying (PSK)
polar
pseudoternary
pulse amplitude modulation (PAM)
pulse code modulation (PCM)
quadrature amplitude modulation (QAM)
quadrature PSK (QPSK)
scrambling
unipolar

Review Questions

5.1. List and briefly define important factors that can be used in evaluating or comparing the various digital-to-digital encoding techniques.
5.2. What is differential encoding?
5.3. Explain the difference between NRZ-L and NRZI.
5.4. Describe two multilevel binary digital-to-digital encoding techniques.
5.5. Define biphase encoding and describe two biphase encoding techniques.
5.6. Explain the function of scrambling in the context of digital-to-digital encoding techniques.
5.7. What function does a modem perform?
5.8. How are binary values represented in amplitude shift keying, and what is the limitation of this approach?
5.9. What is the difference between QPSK and offset QPSK?
5.10. What is QAM?
5.11. What does the sampling theorem tell us concerning the rate of sampling required for an analog signal?
5.12. What are the differences among angle modulation, PM, and FM?

Problems

5.1 Which of the signals of Table 5.2 use differential encoding?
5.2 Develop algorithms for generating each of the codes of Table 5.2 from NRZ-L.
5.3 A modified NRZ code known as enhanced-NRZ (E-NRZ) is sometimes used for high-density magnetic tape recording. E-NRZ encoding entails separating the NRZ-L data stream into 7-bit words; inverting bits 2, 3, 6, and 7; and adding one parity bit to each word. The parity bit is chosen to make the total number of 1s in the 8-bit word an odd count. What are the advantages of E-NRZ over NRZ-L? Any disadvantages?
5.4 Develop a state diagram (finite state machine) representation of pseudoternary coding.
5.5 Consider the following signal encoding technique. Binary data are presented as input, a_m, for $m = 1, 2, 3, \ldots$ Two levels of processing occur. First, a new set of binary numbers are produced:

$$b_0 = 0$$
$$b_m = (a_m + b_{m-1}) \bmod 2$$

These are then encoded as

$$c_m = b_m - b_{m-1}$$

On reception, the original data are recovered by

$$a_m = c_m \bmod 2$$

a. Verify that the received values of a_m equal the transmitted values of a_m.
b. What sort of encoding is this?

5.6 For the bit stream 01001110, sketch the waveforms for each of the codes of Table 5.2. Assume that the signal level for the preceding bit for NRZI was high; the most recent preceding 1 bit (AMI) has a negative voltage; and the most recent preceding 0 bit (pseudoternary) has a negative voltage.
5.7 The waveform of Figure 5.25 belongs to a Manchester encoded binary data stream. Determine the beginning and end of bit periods (i.e., extract clock information) and give the data sequence.

Figure 5.25 A Manchester Stream

5.8 Consider a stream of binary data consisting of a long sequence of 1s followed by a zero followed by a long string of 1s, with the same assumptions as Problem 5.6. Draw the waveform for this sequence using

a. NRZ-L
b. Bipolar-AMI
c. Pseudoternary

5.9 The bipolar-AMI waveform representing the binary sequence 0100101011 is transmitted over a noisy channel. The received waveform is shown in Figure 5.26; it contains a single error. Locate the position of this error and explain your answer.

5.10 One positive side effect of bipolar encoding is that a bipolar violation (two consecutive + pulses or two consecutive − pulses separated by any number of zeros) indicates to the receiver that an error has occurred in transmission. Unfortunately, upon the receipt of such a violation, the receiver does not know which bit is in error (only that an error has occurred). For the received bipolar sequence

$$+ - 0 + - 0 - +$$

which has one bipolar violation, construct two scenarios (each of which involves a different transmitted bit stream with one transmitted bit being converted via an error) that will produce this same received bit pattern.

5.11 Given the bit pattern 01100, encode this data using ASK, BFSK, and BPSK.

5.12 A sine wave is to be used for two different signaling schemes: (a) PSK; (b) QPSK. The duration of a signal element is 10^{-5} s. If the received signal is of the following form:

$$s(t) = 0.005 \sin(2\pi\, 10^6 t + \theta) \text{ volts}$$

and if the measured noise power at the receiver is 2.5×10^{-8} watts, determine the E_b/N_0 (in dB) for each case.

5.13 Derive an expression for baud rate D as a function of bit rate R for QPSK using the digital encoding techniques of Table 5.2.

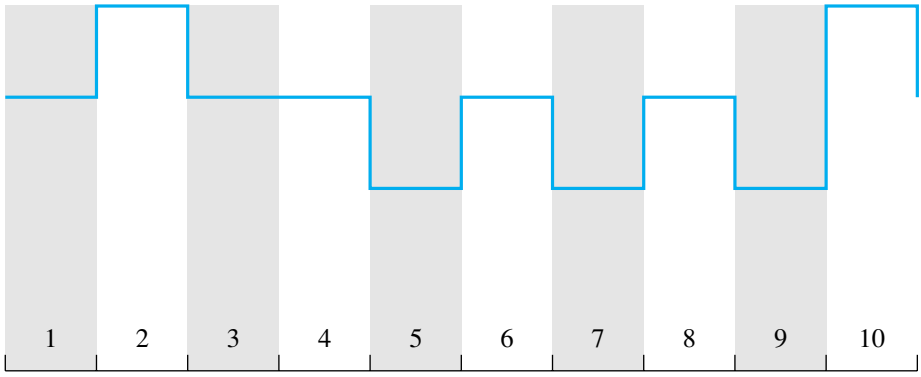


Figure 5.26 A Received Bipolar-AMI Waveform

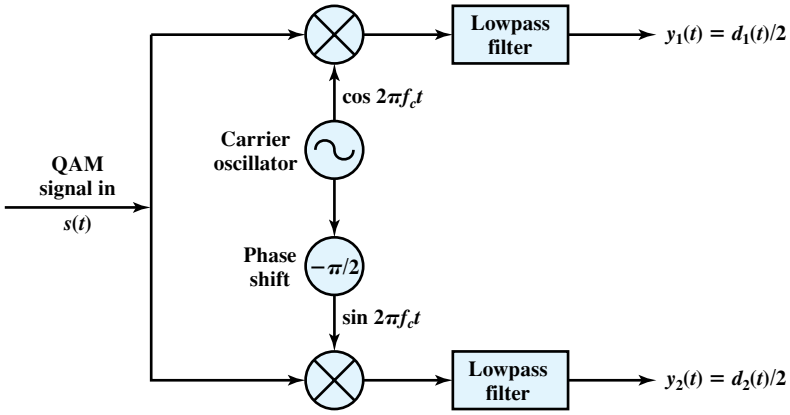


Figure 5.27 QAM Demodulator

5.14 What SNR ratio is required to achieve a bandwidth efficiency of 1.0 for ASK, FSK, PSK, and QPSK? Assume that the required bit error rate is 10^{-6}.

5.15 An NRZ-L signal is passed through a filter with $r = 0.5$ and then modulated onto a carrier. The data rate is 2400 bps. Evaluate the bandwidth for ASK and FSK. For FSK assume that the two frequencies used are 50 kHz and 55 kHz.

5.16 Assume that a telephone line channel is equalized to allow bandpass data transmission over a frequency range of 600 to 3000 Hz. The available bandwidth is 2400 Hz. For $r = 1$, evaluate the required bandwidth for 2400 bps QPSK and 4800-bps, eight-level multilevel signaling. Is the bandwidth adequate?

5.17 Figure 5.27 shows the QAM demodulator corresponding to the QAM modulator of Figure 5.14. Show that this arrangement does recover the two signals $d_1(t)$ and $d_2(t)$, which can be combined to recover the original input.

5.18 Why should PCM be preferable to DM for encoding analog signals that represent digital data?

5.19 Are the modem and the codec functional inverses (i.e., could an inverted modem function as a codec, or vice versa)?

5.20 A signal is quantized using 10-bit PCM. Find the signal-to-quantization noise ratio.

5.21 Consider an audio signal with spectral components in the range 300 to 3000 Hz. Assume that a sampling rate of 7000 samples per second will be used to generate a PCM signal.

a. For SNR = 30 dB, what is the number of uniform quantization levels needed?

b. What data rate is required?

5.22 Find the step size δ required to prevent slope overload noise as a function of the frequency of the highest-frequency component of the signal. Assume that all components have amplitude A.

5.23 A PCM encoder accepts a signal with a full-scale voltage of 10 V and generates 8-bit codes using uniform quantization. The maximum normalized quantized voltage is $1 - 2^{-8}$. Determine (a) normalized step size, (b) actual step size in volts, (c) actual maximum quantized level in volts, (d) normalized resolution, (e) actual resolution, and (f) percentage resolution.

5.24 The analog waveform shown in Figure 5.28 is to be delta modulated. The sampling period and the step size are indicated by the grid on the figure. The first DM output and the staircase function for this period are also shown. Show the rest of the staircase function and give the DM output. Indicate regions where slope overload distortion exists.

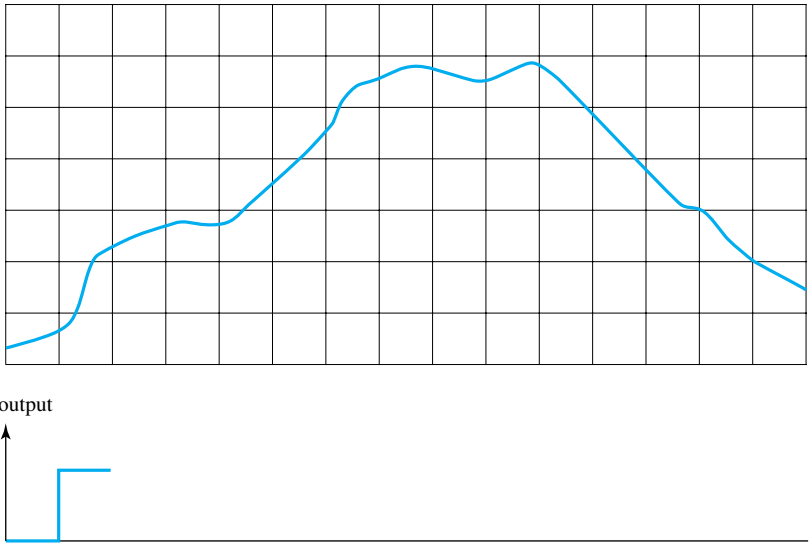


Figure 5.28 Delta Modulation Example

5.25 Consider the angle-modulated signal

$$s(t) = 10\cos[(10^8)\pi t + 5\sin 2\pi(10^3)t]$$

Find the maximum phase deviation and the maximum frequency deviation.

5.26 Consider the angle-modulated signal

$$s(t) = 10\cos[2\pi(10^6)t + 0.1\sin(10^3)\pi t]$$

a. Express $s(t)$ as a PM signal with $n_p = 10$.
b. Express $s(t)$ as an FM signal with $n_f = 10\pi$.

5.27 Let $m_1(t)$ and $m_2(t)$ be message signals and let $s_1(t)$ and $s_2(t)$ be the corresponding modulated signals using a carrier frequency of f_c.

a. Show that if simple AM modulation is used, then $m_1(t) + m_2(t)$ produces a modulated signal equal that is a linear combination of $s_1(t)$ and $s_2(t)$. This is why AM is sometimes referred to as linear modulation.
b. Show that if simple PM modulation is used, then $m_1(t) + m_2(t)$ produces a modulated signal that is not a linear combination of $s_1(t)$ and $s_2(t)$. This is why angle modulation is sometimes referred to as nonlinear modulation.