CHAPTER 9

SPREAD SPECTRUM

All creative people want to do the unexpected.

—Ecstasy and Me: My Life as a Woman, Hedy Lamarr

KEY POINTS

- Spread spectrum is an important form of encoding for wireless communications. The use of spread spectrum makes jamming and interception more difficult and provides improved reception.
- The basic idea of spread spectrum is to modulate the signal so as to increase significantly the bandwidth (spread the spectrum) of the signal to be transmitted.
- **Frequency-hopping spread spectrum** is a form of spread spectrum in which the signal is broadcast over a seemingly random series of radio frequencies, hopping from frequency to frequency at fixed intervals.
- **Direct sequence spread spectrum** is a form of spread spectrum in which each bit in the original signal is represented by multiple bits in the transmitted signal, using a spreading code.
- **Code division multiple access** exploits the nature of spread spectrum transmission to enable multiple users to independently use the same bandwidth with very little interference.

Spread spectrum is an important form of encoding for wireless communications. This technique does not fit neatly into the categories defined in Chapter 5, as it can be used to transmit either analog or digital data, using an analog signal.

The spread spectrum technique was developed initially for military and intelligence requirements. The essential idea is to spread the information signal over a wider bandwidth to make jamming and interception more difficult. The first type of spread spectrum developed is known as frequency hopping.[1] A more recent type of spread spectrum is direct sequence. Both of these techniques are used in various wireless communications standards and products.

After a brief overview, we look at these two spread spectrum techniques. We then examine a multiple access technique based on spread spectrum.

[1]Spread spectrum (using frequency hopping) was invented, believe it or not, by Hollywood screen siren Hedy Lamarr in 1940 at the age of 26. She and a partner who later joined her effort were granted a patent in 1942 (U.S. Patent 2,292,387; 11 August 1942). Lamarr considered this her contribution to the war effort and never profited from her invention.

9.1 THE CONCEPT OF SPREAD SPECTRUM

Figure 9.1 highlights the key characteristics of any spread spectrum system. Input is fed into a channel encoder that produces an analog signal with a relatively narrow bandwidth around some center frequency. This signal is further modulated using a sequence of digits known as a spreading code or spreading sequence. Typically, but not always, the spreading code is generated by a pseudonoise, or pseudorandom number, generator. The effect of this modulation is to increase significantly the bandwidth (spread the spectrum) of the signal to be transmitted. On the receiving end, the same digit sequence is used to demodulate the spread spectrum signal. Finally, the signal is fed into a channel decoder to recover the data.

Several things can be gained from this apparent waste of spectrum:

- The signals gains immunity from various kinds of noise and multipath distortion. The earliest applications of spread spectrum were military, where it was used for its immunity to jamming.
- It can also be used for hiding and encrypting signals. Only a recipient who knows the spreading code can recover the encoded information.
- Several users can independently use the same higher bandwidth with very little interference. This property is used in cellular telephony applications, with a technique know as code division multiplexing (CDM) or code division multiple access (CDMA).

A comment about pseudorandom numbers is in order. These numbers are generated by an algorithm using some initial value called the seed. The algorithm is deterministic and therefore produces sequences of numbers that are not statistically random. However, if the algorithm is good, the resulting sequences will pass many reasonable tests of randomness. Such numbers are often referred to as pseudorandom numbers.[2] The important point is that unless you know the algorithm and the seed, it is impractical to predict the sequence. Hence, only a receiver that shares this information with a transmitter will be able to decode the signal successfully.

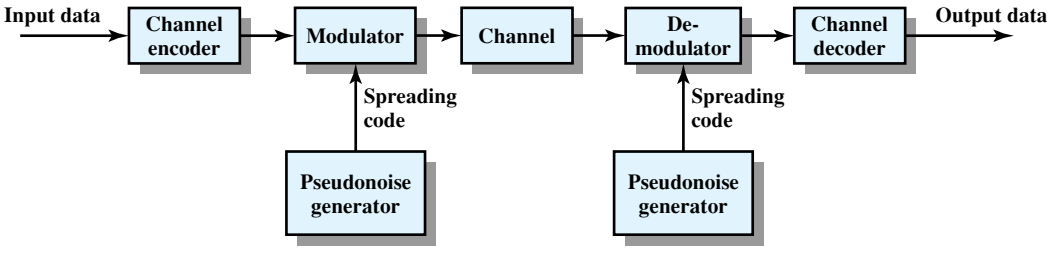

Figure 9.1 General Model of Spread Spectrum Digital Communication System

[2]See [STAL05] for a more detailed discussion of pseudorandom numbers.

9.2 FREQUENCY-HOPPING SPREAD SPECTRUM

With frequency-hopping spread spectrum (FHSS), the signal is broadcast over a seemingly random series of radio frequencies, hopping from frequency to frequency at fixed intervals. A receiver, hopping between frequencies in synchronization with the transmitter, picks up the message. Would-be eavesdroppers hear only unintelligible blips. Attempts to jam the signal on one frequency succeed only at knocking out a few bits of it.

Basic Approach

Figure 9.2 shows an example of a frequency-hopping signal. A number of channels are allocated for the FH signal. Typically, there are 2^k carrier frequencies forming 2^k channels. The spacing between carrier frequencies and hence the width of each channel usually corresponds to the bandwidth of the input signal. The transmitter operates in one channel at a time for a fixed interval; for example, the IEEE 802.11 standard uses a 300-ms interval. During that interval, some number of bits (possibly a fraction of a bit, as discussed subsequently) is transmitted using some encoding scheme. A spreading code dictates the sequence of channels used. Both transmitter and receiver use the same code to tune into a sequence of channels in synchronization.

A typical block diagram for a frequency-hopping system is shown in Figure 9.3. For transmission, binary data are fed into a modulator using some digital-to-analog encoding scheme, such as frequency shift keying (FSK) or binary phase shift keying (BPSK). The resulting signal is centered on some base frequency. A pseudonoise (PN), or pseudorandom number, source serves as an index into a table of frequencies; this is the spreading code referred to previously. Each k bits of the PN source specifies one of the 2^k carrier frequencies. At each successive interval (each k PN bits), a new carrier frequency is selected. This frequency is then modulated by the signal produced from the initial modulator to produce a new signal with the same shape but now centered on the selected carrier frequency. On reception,

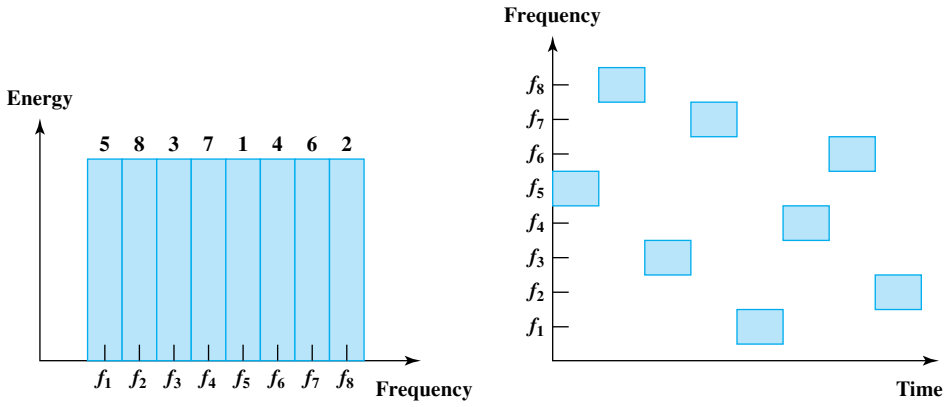

Figure 9.2 Frequency-Hopping Example

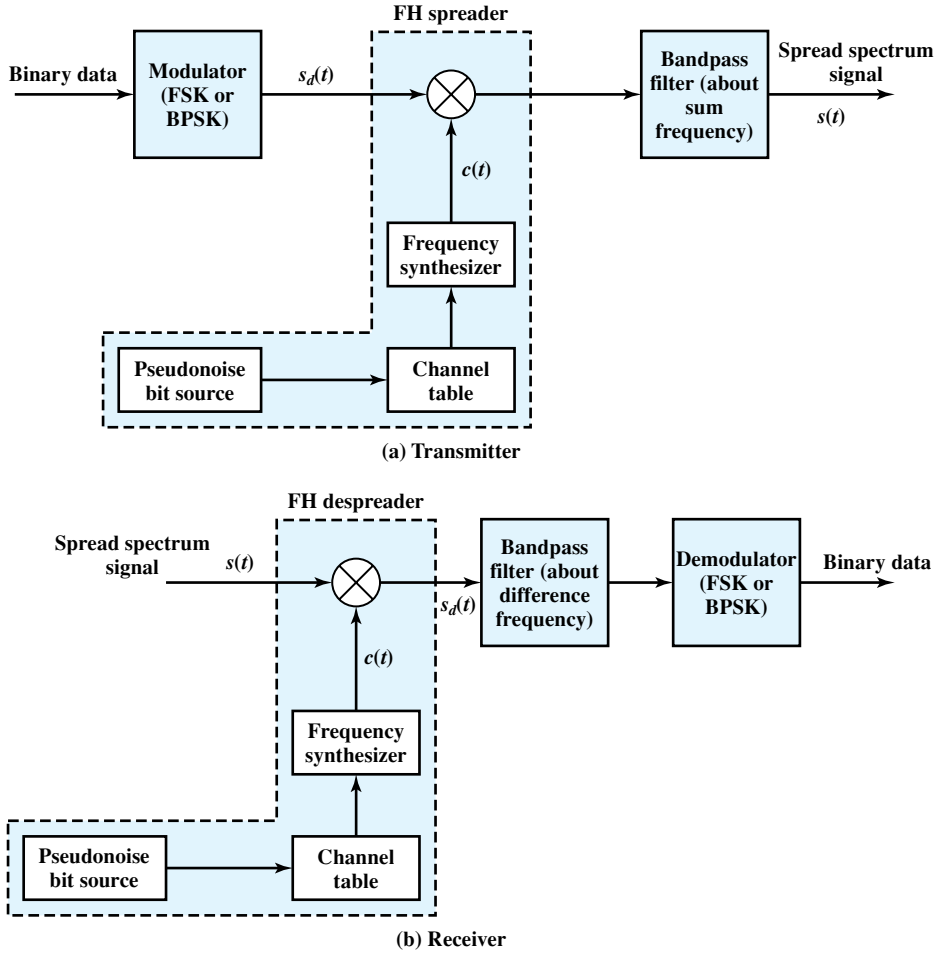

Figure 9.3 Frequency-Hopping Spread Spectrum System

the spread spectrum signal is demodulated using the same sequence of PN-derived frequencies and then demodulated to produce the output data.

Figure 9.3 indicates that the two signals are multiplied. Let us give an example of how this works, using BFSK as the data modulation scheme. We can define the FSK input to the FHSS system as [compare to Equation (5.3)]:

$$s_d(t) = A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f)t) \qquad \text{for } iT < t < (i+1)T \qquad \textbf{(9.1)}$$

where

A = amplitude of signal
f_0 = base frequency
b_i = value of the ith bit of data (+1 for binary 1, −1 for binary 0)
Δf = frequency separation
T = bit duration; data rate = $1/T$

Thus, during the ith bit interval, the frequency of the data signal is f_0 if the data bit is -1 and $f_0 + \Delta f$ if the data bit is $+1$.

The frequency synthesizer generates a constant-frequency tone whose frequency hops among a set of 2^k frequencies, with the hopping pattern determined by k bits from the PN sequence. For simplicity, assume the duration of one hop is the same as the duration of one bit and we ignore phase differences between the data signal $s_d(t)$ and the spreading signal, also called a **chipping signal**, $c(t)$. Then the product signal during the ith hop (during the ith bit) is

$$p(t) = s_d(t)c(t) = A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f)t)\cos(2\pi f_i t)$$

where f_i is the frequency of the signal generated by the frequency synthesizer during the ith hop. Using the trigonometric identity[3] $\cos(x)\cos(y) = (1/2)(\cos(x + y) + \cos(x - y))$, we have

$$p(t) = 0.5A\,[\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f + f_i)t) + \cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f - f_i)t)]$$

A bandpass filter (Figure 9.3) is used to block the difference frequency and pass the sum frequency, yielding an FHSS signal of

$$s(t) = 0.5A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f + f_i)t) \tag{9.2}$$

Thus, during the ith bit interval, the frequency of the data signal is $f_0 + f_i$ if the data bit is -1 and $f_0 + f_i + \Delta f$ if the data bit is $+1$.

At the receiver, a signal of the form $s(t)$ just defined will be received. This is multiplied by a replica of the spreading signal to yield a product signal of the form

$$p(t) = s(t)c(t) = 0.5A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f + f_i)t)\cos(2\pi f_i t)$$

Again using the trigonometric identity, we have

$$p(t) = s(t)c(t) = 0.25A[\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f + f_i + f_i)t) + \cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f)t)]$$

A bandpass filter (Figure 9.3) is used to block the sum frequency and pass the difference frequency, yielding a signal of the form of $s_d(t)$, defined in Equation (9.1):

$$0.25A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f)t)$$

FHSS Using MFSK

A common modulation technique used in conjunction with FHSS is multiple FSK (MFSK). Recall from Chapter 5 that MFSK uses $M = 2^L$ different frequencies to encode the digital input L bits at a time. The transmitted signal is of the form (Equation 5.4):

$$s_i(t) = A\cos 2\pi f_i t, \qquad 1 \le i \le M$$

[3]See the math refresher document at **WilliamStallings.com/StudentSupport.html** for a summary of trigonometric identities.

where

$$f_i = f_c + (2i - 1 - M)f_d$$
f_c = denotes the carrier frequency
f_d = denotes the difference frequency
M = number of different signal elements = 2^L
L = number of bits per signal element

For FHSS, the MFSK signal is translated to a new frequency every T_c seconds by modulating the MFSK signal with the FHSS carrier signal. The effect is to translate the MFSK signal into the appropriate FHSS channel. For a data rate of R, the duration of a bit is $T = 1/R$ seconds and the duration of a signal element is $T_s = LT$ seconds. If T_c is greater than or equal to T_s, the spreading modulation is referred to as slow-frequency-hop spread spectrum; otherwise it is known as fast-frequency-hop spread spectrum.[4] To summarize,

Slow-frequency-hop spread spectrum	$T_c \geq T_s$
Fast-frequency-hop spread spectrum	$T_c < T_s$

Figure 9.4 shows an example of slow FHSS, using the MFSK example from Figure 5.9. Here we have $M = 4$, which means that four different frequencies are used to encode the data input 2 bits at a time. Each signal element is a discrete frequency tone, and the total MFSK bandwidth is $W_d = Mf_d$. We use an FHSS scheme with $k = 2$. That is, there are $4 = 2^k$ different channels, each of width W_d. The total FHSS bandwidth is $W_s = 2^k W_d$. Each 2 bits of the PN sequence is used to select one of the four channels. That channel is held for a duration of two signal elements, or four bits ($T_c = 2T_s = 4T$).

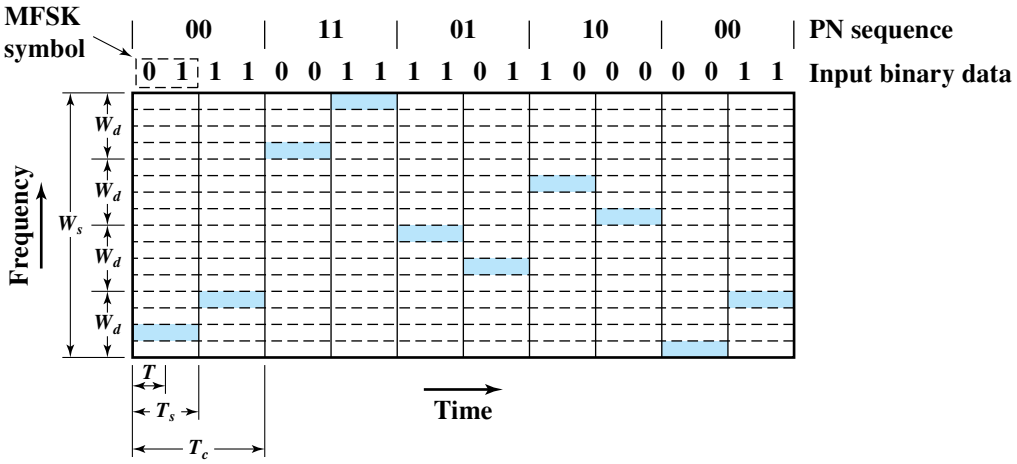

Figure 9.4 Slow Frequency Hop Spread Spectrum Using MFSK ($M = 4, k = 2$)

[4]Some authors use a somewhat different definition (e.g., [PICK82]) of multiple hops per bit for fast frequency hop, multiple bits per hop for slow frequency hop, and one hop per bit if neither fast nor slow. The more common definition, which we use, relates hops to signal elements rather than bits.

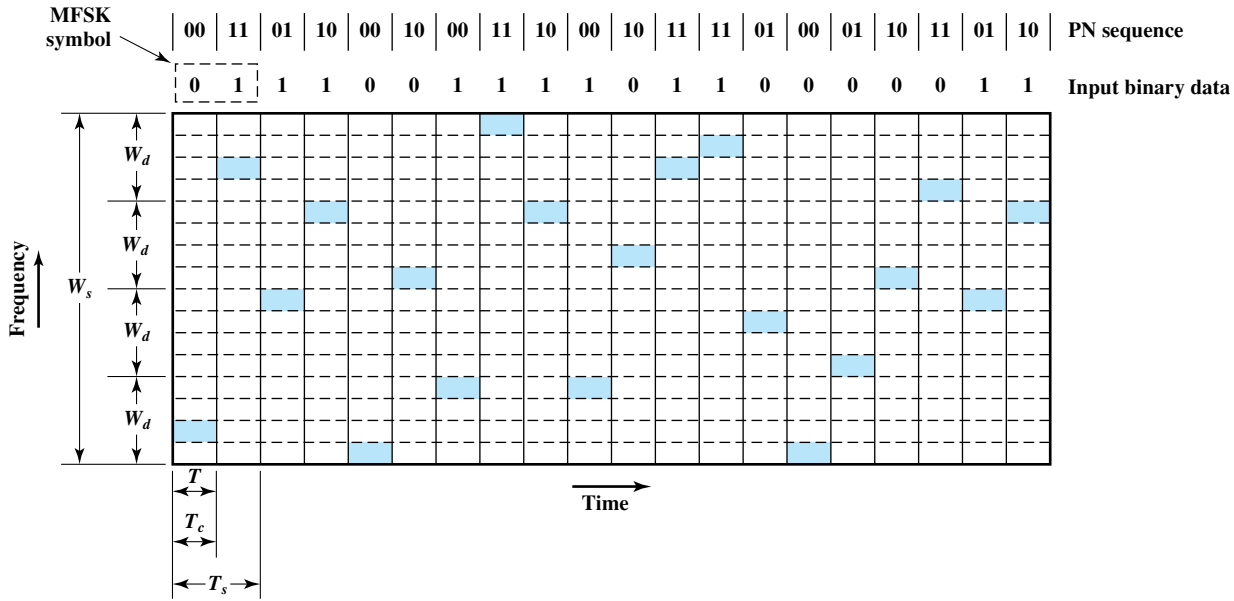

Figure 9.5 Fast Frequency Hop Spread Spectrum Using MFSK ($M = 4, k = 2$)

Figure 9.5 shows an example of fast FHSS, using the same MFSK example. Again, $M = 4$ and $k = 2$. In this case, however, each signal element is represented by two frequency tones. Again, $W_d = Mf_d$ and $W_s = 2^k W_d$. In this example $T_s = 2T_c = 2T$. In general, fast FHSS provides improved performance compared to slow FHSS in the face of noise or jamming. For example, if three or more frequencies (chips) are used for each signal element, the receiver can decide which signal element was sent on the basis of a majority of the chips being correct.

FHSS Performance Considerations

Typically, a large number of frequencies is used in FHSS so that W_s is much larger than W_d. One benefit of this is that a large value of k results in a system that is quite resistant to jamming. For example, suppose we have an MFSK transmitter with bandwidth W_d and a noise jammer of the same bandwidth and fixed power S_j on the signal carrier frequency. Then we have a ratio of signal energy per bit to noise power density per Hertz of

$$\frac{E_b}{N_j} = \frac{E_b W_d}{S_j}$$

If frequency hopping is used, the jammer must jam all 2^k frequencies. With a fixed power, this reduces the jamming power in any one frequency band to $S_j/2^k$. The gain in signal-to-noise ratio, or processing gain, is

$$G_P = 2^k = \frac{W_s}{W_d} \tag{9.3}$$

9.3 DIRECT SEQUENCE SPREAD SPECTRUM

With direct sequence spread spectrum (DSSS), each bit in the original signal is represented by multiple bits in the transmitted signal, using a spreading code. The spreading code spreads the signal across a wider frequency band in direct proportion to the number of bits used. Therefore, a 10-bit spreading code spreads the signal across a frequency band that is 10 times greater than a 1-bit spreading code.

One technique with direct sequence spread spectrum is to combine the digital information stream with the spreading code bit stream using an exclusive-OR (XOR). The XOR obeys the following rules:

$$0 \oplus 0 = 0 \qquad 0 \oplus 1 = 1 \qquad 1 \oplus 0 = 1 \qquad 1 \oplus 1 = 0$$

Figure 9.6 shows an example. Note that an information bit of one inverts the spreading code bits in the combination, while an information bit of zero causes the spreading code bits to be transmitted without inversion. The combination bit stream has the data rate of the original spreading code sequence, so it has a wider bandwidth than the information stream. In this example, the spreading code bit stream is clocked at four times the information rate.

DSSS Using BPSK

To see how this technique works out in practice, assume that a BPSK modulation scheme is to be used. Rather than represent binary data with 1 and 0, it is more

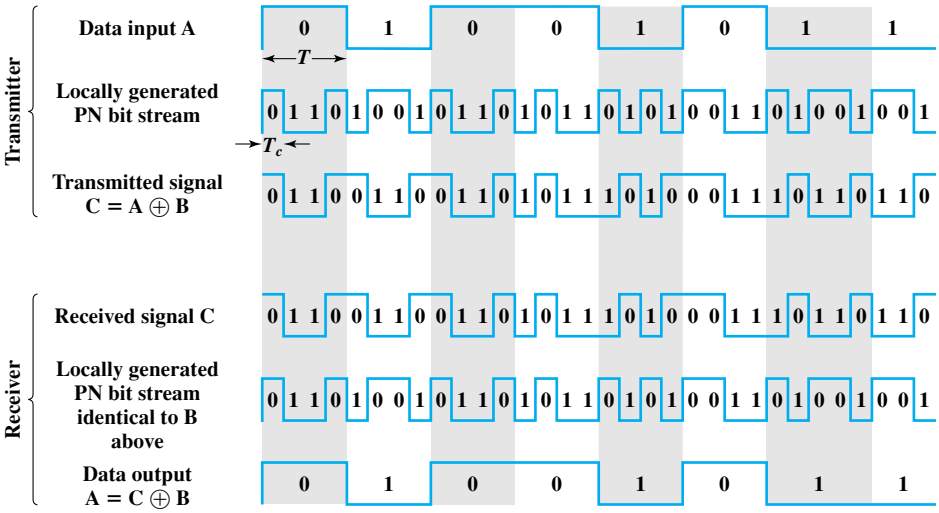

Figure 9.6 Example of Direct Sequence Spread Spectrum

convenient for our purposes to use +1 and −1 to represent the two binary digits. In that case, a BPSK signal can be represented as was shown in Equation (5.6):

$$s_d(t) = Ad(t)\cos(2\pi f_c t) \tag{9.4}$$

where

A = amplitude of signal
f_c = carrier frequency
$d(t)$ = the discrete function that takes on the value of +1 for one bit time if the corresponding bit in the bit stream is 1 and the value of −1 for one bit time if the corresponding bit in the bit stream is 0

To produce the DSSS signal, we multiply the preceding by $c(t)$, which is the PN sequence taking on values of +1 and −1:

$$s(t) = A\,d(t)c(t)\cos(2\pi f_c t) \tag{9.5}$$

At the receiver, the incoming signal is multiplied again by $c(t)$. But $c(t) \times c(t) = 1$ and therefore the original signal is recovered:

$$s(t)c(t) = A\,d(t)c(t)c(t)\cos(2\pi f_c t) = s_d(t)$$

Equation (9.5) can be interpreted in two ways, leading to two different implementations. The first interpretation is to first multiply $d(t)$ and $c(t)$ together and then perform the BPSK modulation. That is the interpretation we have been discussing. Alternatively, we can first perform the BPSK modulation on the data stream $d(t)$ to generate the data signal $s_d(t)$. This signal can then be multiplied by $c(t)$.

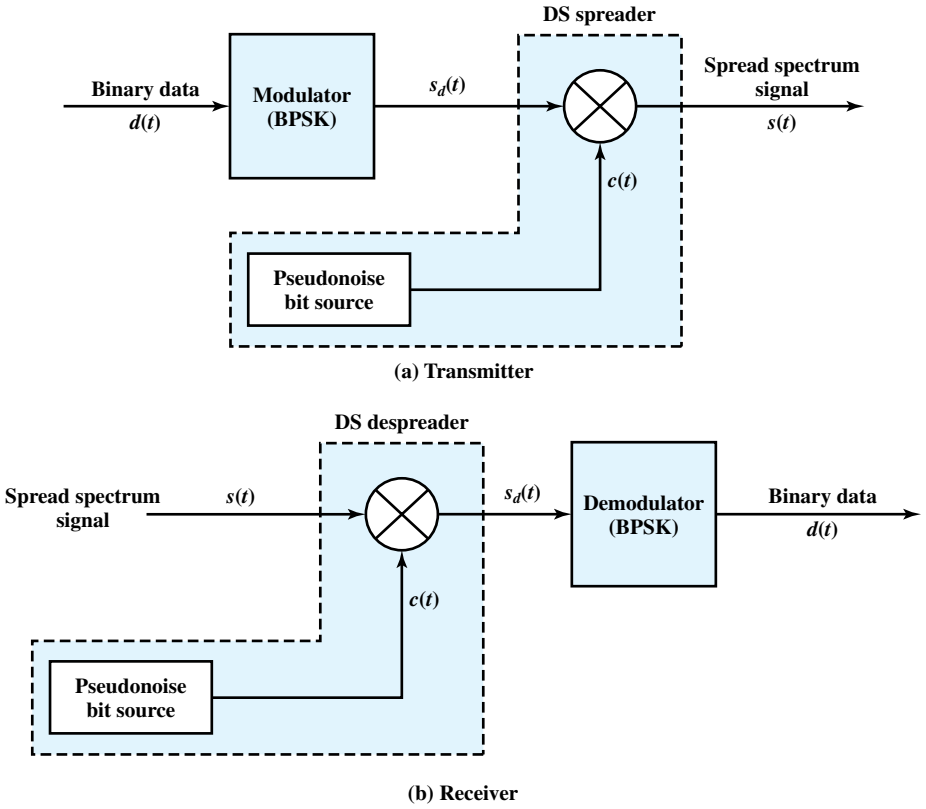

Figure 9.7 Direct Sequence Spread Spectrum System

An implementation using the second interpretation is shown in Figure 9.7. Figure 9.8 is an example of this approach.

DSSS Performance Considerations

The spectrum spreading achieved by the direct sequence technique is easily determined (Figure 9.9). In our example, the information signal has a bit width of T, which is equivalent to a data rate of $1/T$. In that case, the spectrum of the signal, depending on the encoding technique, is roughly $2/T$. Similarly, the spectrum of the PN signal is $2/T_c$. Figure 9.9c shows the resulting spectrum spreading. The amount of spreading that is achieved is a direct result of the data rate of the PN stream.

As with FHSS, we can get some insight into the performance of DSSS by looking at its effectiveness against jamming. Let us assume a simple jamming signal at the center frequency of the DSSS system. The jamming signal has the form

$$s_j(t) = \sqrt{2S_j}\cos(2\pi f_c t)$$

and the received signal is

$$s_r(t) = s(t) + s_j(t) + n(t)$$

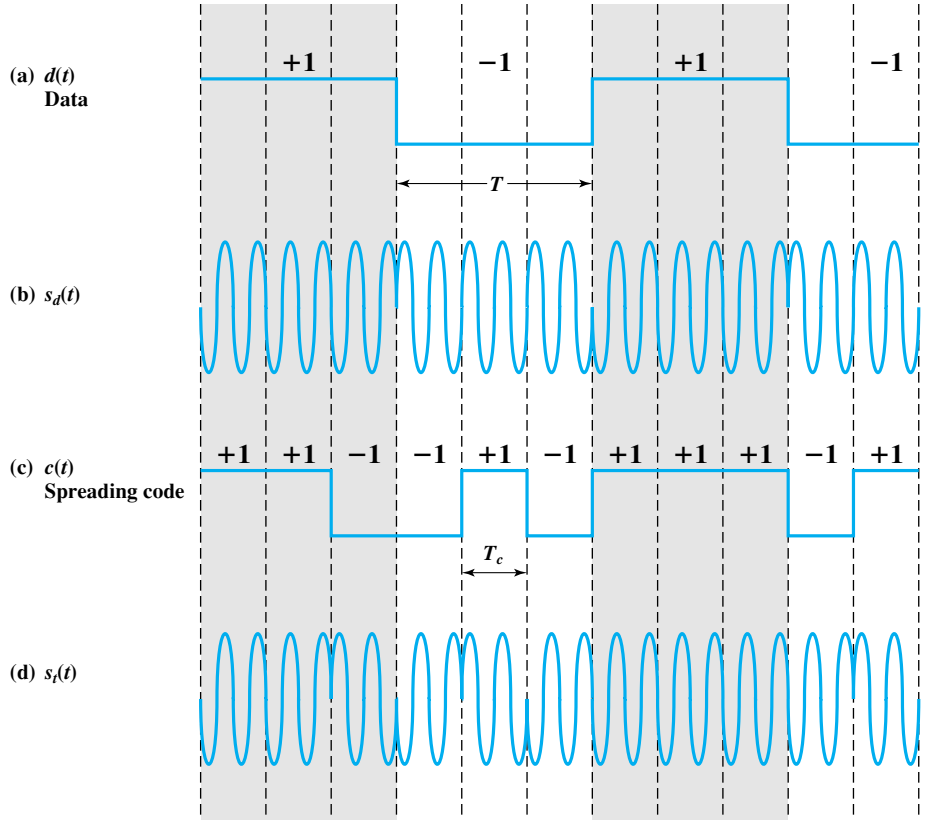

Figure 9.8 Example of Direct-Sequence Spread Spectrum Using BPSK

where

$$\begin{aligned} s(t) &= \text{transmitted signal} \\ s_j(t) &= \text{jamming signal} \\ n(t) &= \text{additive white noise} \\ S_j &= \text{jammer signal power} \end{aligned}$$

The despreader at the receiver multiplies $s_r(t)$ by $c(t)$, so the signal component due to the jamming signal is

$$y_j(t) = \sqrt{2S_j}c(t)\cos(2\pi f_c t)$$

This is simply a BPSK modulation of the carrier tone. Thus, the carrier power S_j is spread over a bandwidth of approximately $2/T_c$. However, the BPSK demodulator (Figure 9.7) following the DSSS despreader includes a bandpass filter matched to the BPSK data, with bandwidth of $2/T$. Thus, most of the jamming power is filtered.

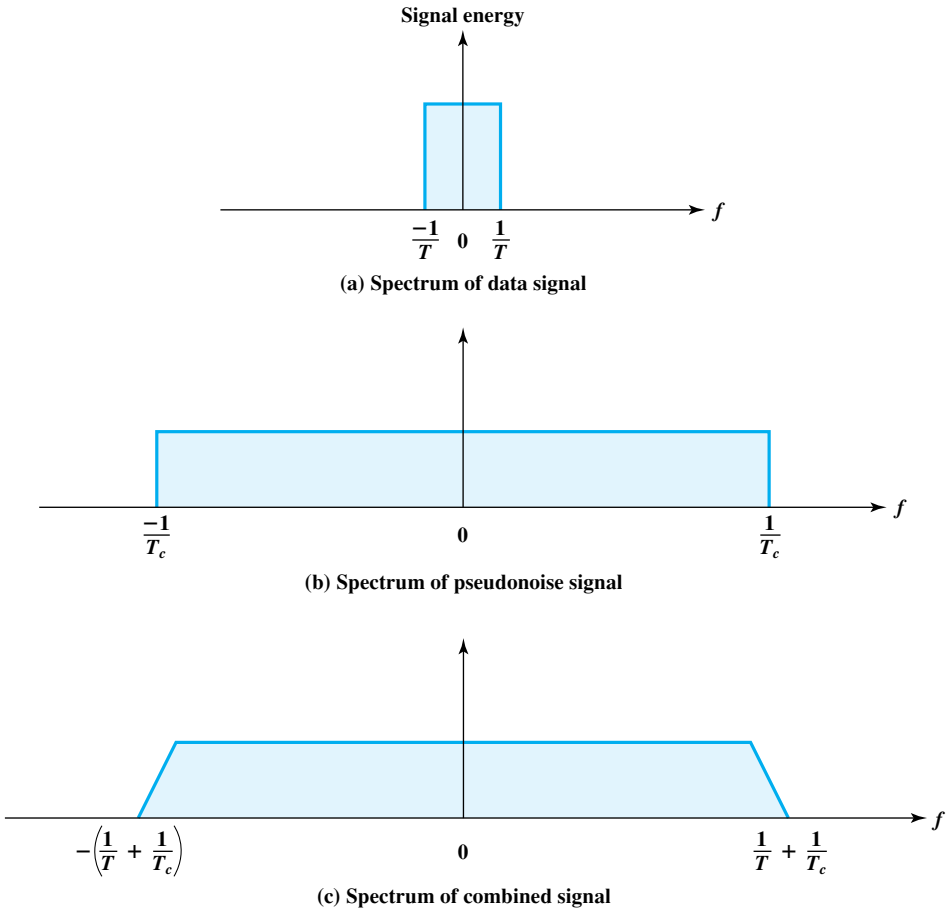

Figure 9.9 Approximate Spectrum of Direct Sequence Spread Spectrum Signal

Although a number of factors come into play, as an approximation, we can say that the jamming power passed by the filter is

$$S_{jF} = S_j(2/T)/(2/T_c) = S_j(T_c/T)$$

The jamming power has been reduced by a factor of (T_c/T) through the use of spread spectrum. The inverse of this factor is the gain in signal-to-noise ratio:

$$G_P = \frac{T}{T_c} = \frac{R_c}{R} \approx \frac{W_s}{W_d} \tag{9.6}$$

where R_c is the spreading bit rate, R is the data rate, W_d is the signal bandwidth, and W_s is the spread spectrum signal bandwidth. The result is similar to the result for FHSS (Equation 9.3).

9.4 CODE DIVISION MULTIPLE ACCESS

Basic Principles

CDMA is a multiplexing technique used with spread spectrum. The scheme works in the following manner. We start with a data signal with rate D, which we call the bit data rate. We break each bit into k chips according to a fixed pattern that is specific to each user, called the user's code. The new channel has a chip data rate of kD chips per second. As an illustration we consider a simple example[5] with $k = 6$. It is simplest to characterize a code as a sequence of 1s and −1s. Figure 9.10 shows the codes for three users, A, B, and C, each of which is communicating with the same base station receiver, R. Thus, the code for user A is $c_A = <1, -1, -1, 1, -1, 1>$. Similarly, user B has code $c_B = <1, 1, -1, -1, 1, 1>$, and user C has $c_C = <1, 1, -1, 1, 1, -1>$.

We now consider the case of user A communicating with the base station. The base station is assumed to know A's code. For simplicity, we assume that communication is already synchronized so that the base station knows when to look for codes. If A wants to send a 1 bit, A transmits its code as a chip pattern $<1, -1, -1, 1, -1, 1>$. If a 0 bit is to be sent, A transmits the complement (1s and −1s reversed) of its code, $<-1, 1, 1, -1, 1, -1>$. At the base station the receiver decodes the chip patterns. In our simple version, if the receiver R receives a chip pattern $d = <d1, d2, d3, d4, d5, d6>$, and the receiver is seeking to communicate

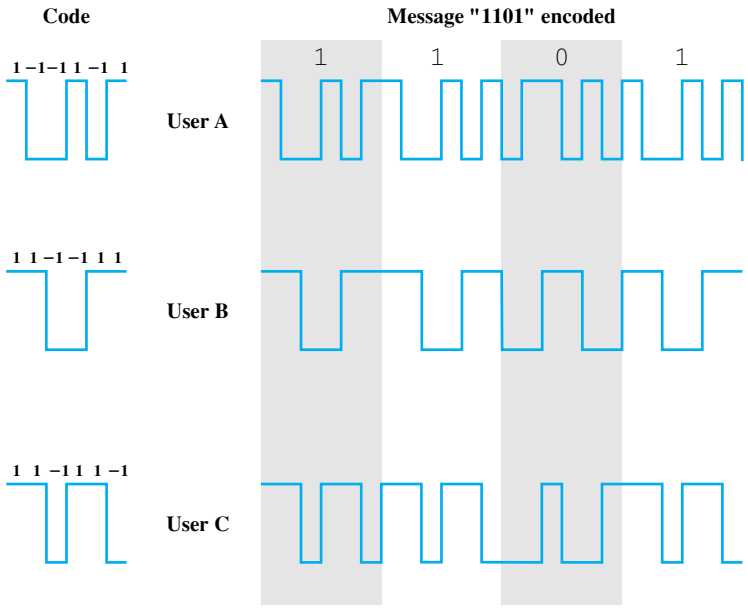

Figure 9.10 CDMA Example

[5]This example was provided by Professor Richard Van Slyke of the Polytechnic University of Brooklyn.

with a user u so that it has at hand u's code, $<c1, c2, c3, c4, c5, c6>$, the receiver performs electronically the following decoding function:

$$S_u(d) = d1 \times c1 + d2 \times c2 + d3 \times c3 + d4 \times c4 + d5 \times c5 + d6 \times c6$$

The subscript u on S simply indicates that u is the user that we are interested in. Let's suppose the user u is actually A and see what happens. If A sends a 1 bit, then d is $<1, -1, -1, 1, -1, 1>$ and the preceding computation using S_A becomes

$$S_A(1, -1, -1, 1, -1, 1) = 1 \times 1 + (-1) \times (-1) + (-1) \times (-1) + 1 \times 1 + (-1) \times (-1) + 1 \times 1 = 6$$

If A sends a 0 bit that corresponds to $d = <-1, 1, 1, -1, 1, -1>$, we get

$$S_A(-1, 1, 1, -1, 1, -1) = -1 \times 1 + 1 \times (-1) + 1 \times (-1) + (-1) \times 1 + 1 \times (-1) + (-1) \times 1 = -6$$

Please note that it is always the case that $-6 \leq S_A(d) \leq 6$ no matter what sequence of -1s and 1s that d is, and that the only d's resulting in the extreme values of 6 and -6 are A's code and its complement, respectively. So if S_A produces a $+6$, we say that we have received a 1 bit from A; if S_A produces a -6, we say that we have received a 0 bit from user A; otherwise, we assume that someone else is sending information or there is an error. So why go through all this? The reason becomes clear if we see what happens if user B is sending and we try to receive it with S_A, that is, we are decoding with the wrong code, A's. If B sends a 1 bit, then $d = <1, 1, -1, -1, 1, 1>$. Then

$$S_A(1, 1, -1, -1, 1, 1) = 1 \times 1 + 1 \times (-1) + (-1) \times (-1) + (-1) \times 1 + 1 \times (-1) + 1 \times 1 = 0$$

Thus, the unwanted signal (from B) does not show up at all. You can easily verify that if B had sent a 0 bit, the decoder would produce a value of 0 for S_A again. This means that if the decoder is linear and if A and B transmit signals s_A and s_B, respectively, at the same time, then $S_A(s_A + s_B) = S_A(s_A) + S_A(s_B) = S_A(s_A)$ since the decoder ignores B when it is using A's code. The codes of A and B that have the property that $S_A(c_B) = S_B(c_A) = 0$ are called **orthogonal**.[6] Such codes are very nice to have but there are not all that many of them. More common is the case when $S_X(c_Y)$ is small in absolute value when $X \neq Y$. Then it is easy to distinguish between the two cases when $X = Y$ and when $X \neq Y$. In our example $S_A(c_C) = S_c(c_A) = 0$, but $S_B(c_C) = S_C(c_B) = 2$. In the latter case the C signal would make a small contribution to the decoded signal instead of 0. Using the decoder, S_u, the receiver can sort out transmission from u even when there may be other users broadcasting in the same cell.

Table 9.1 summarizes the example from the preceding discussion.

In practice, the CDMA receiver can filter out the contribution from unwanted users or they appear as low-level noise. However, if there are many users competing for the channel with the user the receiver is trying to listen to, or if the signal power of one or more competing signals is too high, perhaps because it is very near the receiver (the "near/far" problem), the system breaks down.

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[6]See Appendix J for a discussion of orthogonality of chipping codes.

Table 9.1 CDMA Example

(a) User's codes

User A	1	−1	−1	1	−1	1
User B	1	1	−1	−1	1	1
User C	1	1	−1	1	1	−1

(b) Transmission from A

Transmit (data bit = 1)	1	−1	−1	1	−1	1	
Receiver codeword	1	−1	−1	1	−1	1	
Multiplication	1	1	1	1	1	1	= 6

Transmit (data bit = 0)	−1	1	1	−1	1	−1	
Receiver codeword	1	−1	−1	1	−1	1	
Multiplication	−1	−1	−1	−1	−1	−1	= −6

(c) Transmission from B, receiver attempts to recover A's transmission

Transmit (data bit = 1)	1	1	−1	−1	1	1	
Receiver codeword	1	−1	−1	1	−1	1	
Multiplication	1	−1	1	−1	−1	1	= 0

(d) Transmission from C, receiver attempts to recover B's transmission

Transmit (data bit = 1)	1	1	−1	1	1	−1	
Receiver codeword	1	1	−1	−1	1	1	
Multiplication	1	1	1	−1	1	−1	= 2

(e) Transmission from B and C, receiver attempts to recover B's transmission

B (data bit = 1)	1	1	−1	−1	1	1	
C (data bit = 1)	1	1	−1	1	1	−1	
Combined signal	2	2	−2	0	2	0	
Receiver codeword	1	1	−1	−1	1	1	
Multiplication	2	2	2	0	2	0	= 8

CDMA for Direct Sequence Spread Spectrum

Let us now look at CDMA from the viewpoint of a DSSS system using BPSK. Figure 9.11 depicts a configuration in which there are n users, each transmitting using a different, orthogonal, PN sequence (compare Figure 9.7). For each user, the data stream to be transmitted, $d_i(t)$, is BPSK modulated to produce a signal with a bandwidth of W_s and then multiplied by the spreading code for that user, $c_i(t)$. All of the signals, plus noise, are received at the receiver's antenna. Suppose that the receiver is attempting to recover the data of user 1. The incoming signal is multiplied by the spreading code of user 1 and then demodulated. The effect of this is to narrow the bandwidth of that portion of the incoming signal corresponding to user 1 to the original bandwidth of the unspread signal, which is proportional to the data rate. Incoming signals from

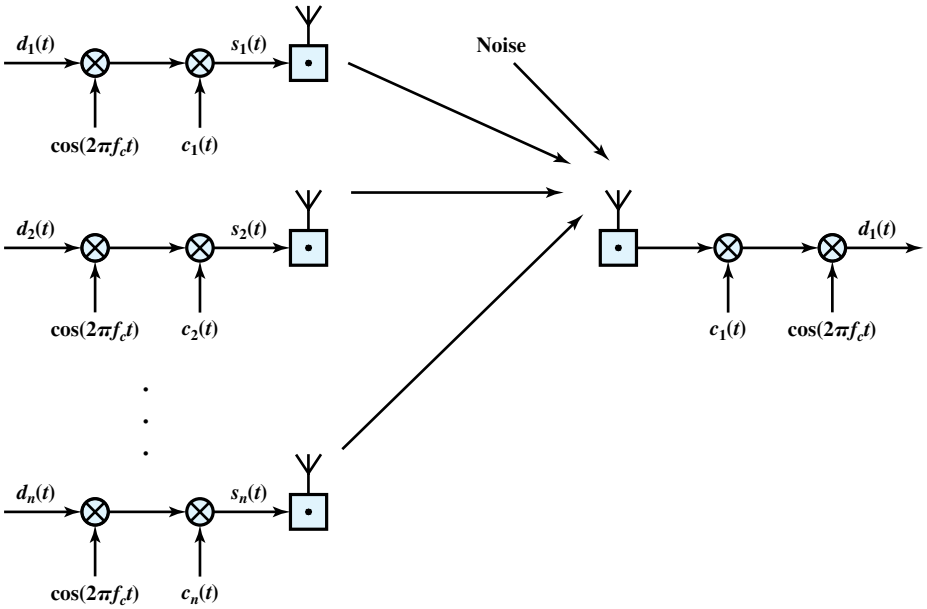

Figure 9.11 CDMA in a DSSS Environment

other users are not despread by the spreading code from user 1 and hence retain their bandwidth of W_s. Thus the unwanted signal energy remains spread over a large bandwidth and the wanted signal is concentrated in a narrow bandwidth. The bandpass filter at the demodulator can therefore recover the desired signal.

9.5 RECOMMENDED READING AND WEB SITE

[DIXO94] provides comprehensive treatment of spread spectrum. [TANT98] contains reprints of many important papers in the field, including [PICK82], which provides an excellent introduction to spread spectrum.

DIXO94 Dixon, R. *Spread Spectrum Systems with Commercial Applications.* New York: Wiley, 1994.

PICK82 Pickholtz, R.; Schilling, D.; and Milstein, L. "Theory of Spread Spectrum Communications—A Tutorial." *IEEE Transactions on Communications*, May 1982. Reprinted in [TANT98].

TANT98 Tantaratana, S, and Ahmed, K., eds. *Wireless Applications of Spread Spectrum Systems: Selected Readings.* Piscataway, NJ: IEEE Press, 1998.

Recommended Web site:

- **Spread Spectrum Scene:** Excellent source of information and links

9.6 KEY TERMS, REVIEW QUESTIONS, AND PROBLEMS

Key Terms

chip	fast FHSS	slow FHSS
chipping signal	frequency-hopping spread spectrum (FHSS)	spread spectrum
code division multiple access (CDMA)	orthogonal	spreading code
direct sequence spread spectrum (DSSS)	pseudonoise (PN)	spreading sequence

Review Questions

9.1. What is the relationship between the bandwidth of a signal before and after it has been encoded using spread spectrum?

9.2. List three benefits of spread spectrum.

9.3. What is frequency-hopping spread spectrum?

9.4. Explain the difference between slow FHSS and fast FHSS.

9.5. What is direct sequence spread spectrum?

9.6. What is the relationship between the bit rate of a signal before and after it has been encoded using DSSS?

9.7. What is CDMA?

Problems

9.1 Assume we wish to transmit a 56-kbps data stream using spread spectrum.

a. Find the channel bandwidth required to achieve a 56-kbps channel capacity when SNR = 0.1, 0.01, and 0.001.

b. In an ordinary (not spread spectrum) system, a reasonable goal for bandwidth efficiency might be 1 bps/Hz. That is, to transmit a data stream of 56 kbps, a bandwidth of 56 kHz is used. In this case, what is the minimum SNR that can be endured for transmission without appreciable errors? Compare to the spread spectrum case.
Hint: Review the discussion of channel capacity in Section 3.4.

9.2 An FHSS system employs a total bandwidth of $W_s = 400$ MHz and an individual channel bandwidth of 100 Hz. What is the minimum number of PN bits required for each frequency hop?

9.3 An FHSS system using MFSK with $M = 4$ employs 1000 different frequencies. What is the processing gain?

9.4 The following table illustrates the operation of an FHSS system for one complete period of the PN sequence.

Time	0	1	2	3	4	5	6	7	8	9	10	11
Input data	0	1	1	1	1	1	1	0	0	0	1	0
Frequency	f_1		f_3		f_{23}		f_{22}		f_8		f_{10}	
PN sequence	001				110				011			

Time	12	13	14	15	16	17	18	19
Input data	0	1	1	1	1	0	1	0
Frequency	f_1		f_3		f_2		f_2	
PN sequence	001				001			

a. What is the period of the PN sequence, in terms of bits in the sequence?
b. The system makes use of a form of FSK. What form of FSK is it?
c. What is the number of bits per signal element?
d. What is the number of FSK frequencies?
e. What is the length of a PN sequence per hop?
f. Is this a slow or fast FH system?
g. What is the total number of possible carrier frequencies?
h. Show the variation of the base, or demodulated, frequency with time.

9.5 The following table illustrates the operation of a FHSS system using the same PN sequence as Problem 9.4.

Time	0	1	2	3	4	5	6	7	8	9	10	11
Input data	0	1	1	1	1	1	1	0	0	0	1	0
Frequency	f_1	f_{21}	f_{11}	f_3	f_3	f_3	f_{22}	f_{10}	f_0	f_0	f_2	f_{22}
PN sequence	001	110	011	001	001	001	110	011	001	001	001	110

Time	12	13	14	15	16	17	18	19
Input data	0	1	1	1	1	0	1	0
Frequency	f_9	f_1	f_3	f_3	f_{22}	f_{10}	f_2	f_2
PN sequence	011	001	001	001	110	011	001	001

a. What is the period of the PN sequence?
b. The system makes use of a form of FSK. What form of FSK is it?
c. What is the number of bits per signal element?
d. What is the number of FSK frequencies?
e. What is the length of a PN sequence per hop?
f. Is this a slow or fast FH system?
g. What is the total number of possible carrier frequencies?
h. Show the variation of the base, or demodulated, frequency with time.

9.6 Consider an MFSK scheme with $f_c = 250$ kHz, $f_d = 25$ kHz, and $M = 8$ ($L = 3$ bits).
a. Make a frequency assignment for each of the eight possible 3-bit data combinations.
b. We wish to apply FHSS to this MFSK scheme with $k = 2$; that is, the system will hop among four different carrier frequencies. Expand the results of part (a) to show the $4 \times 8 = 32$ frequency assignments.

9.7 Figure 9.12, based on one in [BELL00], depicts a simplified scheme for CDMA encoding and decoding. There are seven logical channels, all using DSSS with a spreading code of 7 bits. Assume that all sources are synchronized. If all seven sources transmit a data bit, in the form of a 7-bit sequence, the signals from all sources combine at the receiver so that two positive or two negative values reinforce and a positive and negative value cancel. To decode a given channel, the receiver multiplies the incoming composite signal by the spreading code for that channel, sums the result, and assigns binary 1 for a positive value and binary 0 for a negative value.

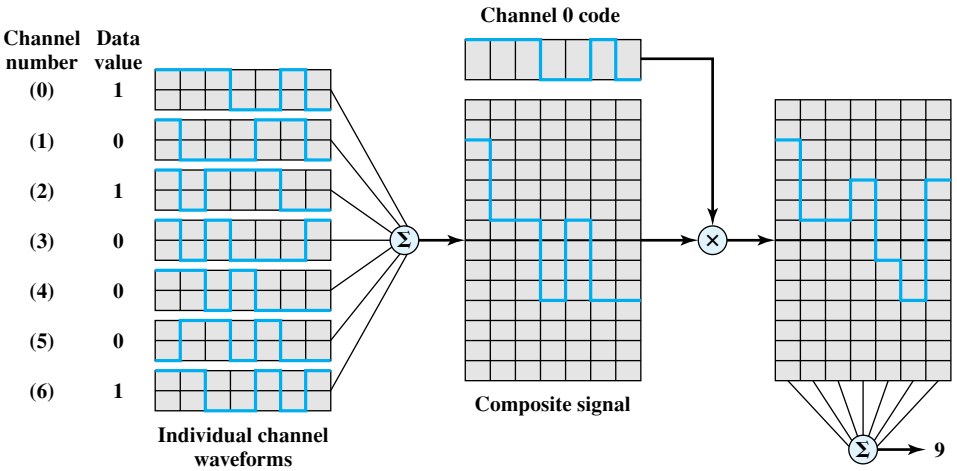

Figure 9.12 Example Seven-Channel CDMA Encoding and Decoding

a. What are the spreading codes for the seven channels?
b. Determine the receiver output measurement for channel 1 and the bit value assigned.
c. Repeat part (b) for channel 2.

9.8 By far, the most widely used technique for pseudorandom number generation is the linear congruential method. The algorithm is parameterized with four numbers, as follows:

m	the modulus	$m > 0$
a	the multiplier	$0 \leq a < m$
c	the increment	$0 \leq c < m$
X_0	the starting value, or seed	$0 \leq X_0 < m$

The sequence of pseudorandom numbers $\{X_n\}$ is obtained via the following iterative equation:

$$X_{n+1} = (aX_n + c) \bmod m$$

If m, a, c, and X_0 are integers, then this technique will produce a sequence of integers with each integer in the range $0 \leq X_n < m$. An essential characteristic of a pseudorandom number generator is that the generated sequence should appear random. Although the sequence is not random, because it is generated deterministically, there is a variety of statistical tests that can be used to assess the degree to which a sequence exhibits randomness. Another desirable characteristic is that the function should be a full-period generating function. That is, the function should generate all the numbers between 0 and m before repeating.

With the linear congruential algorithm, a choice of parameters that provides a full period does not necessarily provide a good randomization. For example, consider the two generators:

$$X_{n+1} = (6X_n) \bmod 13$$
$$X_{n+1} = (7X_n) \bmod 13$$

Write out the two sequences to show that both are full period. Which one appears more random to you?

9.9 We would like m to be very large so that there is the potential for producing a long series of distinct random numbers. A common criterion is that m be nearly equal to the maximum representable nonnegative integer for a given computer. Thus, a value of m near to or equal to 2^{31} is typically chosen. Many experts recommend a value of $2^{31} - 1$. You may wonder why one should not simply use 2^{31}, because this latter number can be represented with no additional bits, and the mod operation should be easier to perform. In general, the modulus $2^k - 1$ is preferable to 2^k. Why is this so?

9.10 In any use of pseudorandom numbers, whether for encryption, simulation, or statistical design, it is dangerous to trust blindly the random number generator that happens to be available in your computer's system library. [PARK88] found that many contemporary textbooks and programming packages make use of flawed algorithms for pseudorandom number generation. This exercise will enable you to test your system.

The test is based on a theorem attributed to Ernesto Cesaro (see [KNUT98] for a proof), which states that the probability is equal to $\frac{6}{\pi^2}$ that the greatest common divisor of two randomly chosen integers is 1. Use this theorem in a program to determine statistically the value of π. The main program should call three subprograms: the random number generator from the system library to generate the random integers; a subprogram to calculate the greatest common divisor of two integers using Euclid's algorithm; and a subprogram that calculates square roots. If these latter two programs are not available, you will have to write them as well. The main program should loop through a large number of random numbers to give an estimate of the aforementioned probability. From this, it is a simple matter to solve for your estimate of π.

If the result is close to 3.14, congratulations! If not, then the result is probably low, usually a value of around 2.7. Why would such an inferior result be obtained?