



Institut Informatika & Bisnis  
**DARMAJAYA**  
Keunggulan. Afiliasi. Mutakhir.



**Kampus  
Merdeka**  
INDONESIA JAYA

**MERDEKA  
BELAJAR**

# Mathematics for Data Science

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PERTEMUAN 1

Egi Safitri, S.Mat., M.Si

[egisafitri@darmajaya.ac.id](mailto:egisafitri@ darmajaya.ac.id)

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## Introduction

### Matrices, Vectors : Addition and Scalar Multiplication

Linear Systems, a Major  
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## Problem Set

**Linear algebra** is a fairly extensive subject that covers vectors and matrices, determinants, systems of linear equations, vector spaces and linear transformations, eigenvalue problems, and other topics. As an area of study it has a broad appeal in that it has many applications in engineering, physics, geometry, computer science, economics, and other areas. It also contributes to a deeper understanding of mathematics itself.





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**Matrices**, which are rectangular arrays of numbers or functions, and vectors are the main tools of linear algebra. Matrices are important because they let us express large amounts of data and functions in an organized and concise form. Furthermore, since matrices are single objects, we denote them by single letters and calculate with them directly. All these features have made matrices and vectors very popular for expressing scientific and mathematical ideas.





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A **matrix** is a rectangular array of numbers or functions which we will enclose in brackets. For example,

$$\begin{bmatrix} 0.3 & 1 & -5 \\ 0 & -0.2 & 16 \end{bmatrix}, \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad (2.1)$$

$$\begin{bmatrix} e^{-2} & 2x^2 \\ e^{6x} & 4x \end{bmatrix}, [ a_1 \quad a_2 \quad a_3 ], \begin{bmatrix} 4 \\ \frac{1}{2} \end{bmatrix} \quad (2.2)$$

are matrices.



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We are given a system of linear equations, briefly a **linear system**, such as :

$$4x_1 + 6x_2 + 9x_3 = 6$$

$$6x_1 - 2x_3 = 20$$

$$5x_1 - 8x_2 + x_3 = 10$$

where  $x_1, x_2, x_3$  are the **unknowns**. We form the **coefficient matrix**, call it **A**, by listing the coefficients of the unknowns in the position in which they appear in the linear equations. In the second equation, there is no unknown  $x_2$  which means that the coefficient of is 0 and hence in matrix **A**,  $a_{22} = 0$ , Thus :



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$$\mathbf{A} = \begin{bmatrix} 4 & 6 & 9 \\ 6 & 0 & -2 \\ 5 & -8 & 1 \end{bmatrix}$$

We form another matrix  $\begin{bmatrix} 4 & 6 & 9 & 6 \\ 6 & 0 & -2 & 20 \\ 5 & -8 & 1 & 10 \end{bmatrix}$

by augmenting  $\mathbf{A}$  with the right sides of the linear system and call it the augmented matrix of the system.



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Let us formalize what we just have discussed. We shall denote matrices by capital boldface letter **A, B, C, . . .**, or by writing the general entry in brackets; thus  $\mathbf{A} = [a_{ij}]$ , and so on.





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$$\mathbf{A} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad (2.3)$$





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A vector is a matrix with only one row or column. Its entries are called the components of the vector. We shall denote vectors by lowercase boldface letters  $\mathbf{a}, \mathbf{b}, \dots$  or by its general component in brackets,  $\mathbf{a} = [a_{ij}]$ , and so on. Our special vectors in (1) suggest that a (general) **row vector** is of the form





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$$\mathbf{a} = [ a_1 \quad a_2 \quad \dots \quad a_n ] .$$

For Instance,  $\mathbf{a} = [ -2 \quad 5 \quad 0.8 \quad 0 \quad 1 ]$





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A **column vector** is of the form ;

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \quad \text{For Instance : } \mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 7 \\ -4 \end{bmatrix}$$



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### Definition (Equality of Matrices)

Two matrices  $\mathbf{A} = [a_{ij}]$  and  $\mathbf{B} = [b_{ij}]$  are equal, written  $\mathbf{A} = \mathbf{B}$ , if and only if they have the same size and the corresponding entries are equal, that is,  $a_{11} = b_{11}, a_{12} = b_{12}$  and so on. Matrices that are not equal are called **different**. Thus, matrices of different sizes are always different.



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### Definition (Addition of Matrices)

The **sum** of two matrices  $\mathbf{A} = [a_{ij}]$  and  $\mathbf{B} = [b_{ij}]$  **of the same size** is written  $\mathbf{A} + \mathbf{B}$  and has entries  $a_{ij} + b_{ij}$  tained by adding the corresponding entries of  $\mathbf{A}$  and  $\mathbf{B}$ . Matrices of different sizes cannot be added.



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### Thus

As a special case, the **sum  $\mathbf{a} + \mathbf{b}$**  of two row vectors or two column vectors, which must have the same number of components, is obtained by adding the corresponding components.



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### Definition (Scalar Multiplication (Multiplication by a Number))

The **product** of any  $m \times n$  matrix  $\mathbf{A} = [a_{ij}]$  and any **scalar**  $c$  (number  $c$ ) is written  $c\mathbf{A}$  and is the  $m \times n$  matrix  $c\mathbf{A} = [ca_{ij}]$  obtained by multiplying each entry of  $\mathbf{A}$ .



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### Then

Here  $(-1)\mathbf{A}$  is simply written  $-\mathbf{A}$  and is called the **negative** of  $\mathbf{A}$ . Similarly,  $(-k)\mathbf{A}$  is written  $-k\mathbf{A}$ . Also  $\mathbf{A} + (-\mathbf{B})$  is written  $\mathbf{A}-\mathbf{B}$  and is called the difference of  $\mathbf{A}$  and  $\mathbf{B}$  (which must have the same size!).





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## Thus

Here  $(-1)\mathbf{A}$  is simply written  $-\mathbf{A}$  and is called the **negative** of  $\mathbf{A}$ . Similarly,  $(-k)\mathbf{A}$  is written  $-k\mathbf{A}$ . Also  $\mathbf{A} + (-\mathbf{B})$  is written  $\mathbf{A}-\mathbf{B}$  and is called the difference of  $\mathbf{A}$  and  $\mathbf{B}$  (which must have the same size!).



# Rules for Matrix Addition and Scalar Multiplication.

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From the familiar laws for the addition of numbers we obtain similar laws for the addition of matrices of the same size  $m \times n$ , namely



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From the familiar laws for the addition of numbers we obtain similar laws for the addition of matrices of the same size  $m \times n$ , namely

1  $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

2  $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$

3  $\mathbf{A} + \mathbf{0} = \mathbf{A}$

4  $\mathbf{A} + (-\mathbf{A}) = \mathbf{0}$



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- 1  $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
- 2  $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$
- 3  $\mathbf{A} + \mathbf{0} = \mathbf{A}$
- 4  $\mathbf{A} + (-\mathbf{A}) = \mathbf{0}$

Here  $\mathbf{0}$  denotes the **zero matrix**(of size  $m \times n$ ), that is the  $m \times n$  matrix with all entries zero. If  $m = 1$  or  $n = 1$ , this is a vector, called a **zero vector**



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Hence matrix addition is **commutative** and **associative**.  
Similarly, for scalar multiplication we obtain the rules :

- 1  $c(\mathbf{A} + \mathbf{B}) = c\mathbf{A} + c\mathbf{B}$
- 2  $(c + k)\mathbf{A} = c\mathbf{A} + k\mathbf{A}$
- 3  $c(k\mathbf{A}) = (ck)\mathbf{A}$
- 4  $1\mathbf{A} = \mathbf{A}$





# Problem Set!

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$$\text{Let : } \mathbf{A} = \begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 5 & 2 \\ 5 & 3 & 4 \\ -2 & 4 & -2 \end{bmatrix}$$
$$\mathbf{C} = \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 0 & 3 \\ 3 & 4 \\ 3 & -1 \end{bmatrix},$$
$$\mathbf{u} = \begin{bmatrix} 1.5 \\ 0 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} -5 \\ -30 \\ 10 \end{bmatrix}$$





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Find the following expressions, indicating which of the rules before, give reasons why they are not defined.

1  $2\mathbf{A} + 4\mathbf{B}, 4\mathbf{B} + 2\mathbf{A}$

2  $4(3\mathbf{A}), 2(5\mathbf{D} + 4\mathbf{C})$

3  $(\mathbf{C} + \mathbf{D}) + \mathbf{E}, (\mathbf{D} + \mathbf{E}) + \mathbf{C}$

4  $(\mathbf{u} + \mathbf{v}) - \mathbf{w}$



Thank You.

Egi Safitri, S.Mat., M.Si

