Mathematics for Data Science

SSD23402
Chapter 2

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Matrix Multiplication

Matrix multiplication means that one multiplies matrices by matrices. Its definition is standard but it looks artificial. *Thus you have to study matrix multiplication carefully*, multiply a few matrices together for practice until you can understand how to do it. Here then is the definition.

Matrix Multiplication by Matrix

Definition (Multiplication of a Matrix by a Matrix)

The **product C=AB** (in this order) of an $m \times n$ matrix **A** $= a_{jk}$ times an $r \times p$ matrix **B** $= a_{jk}$ is defined if and only if $r = n$ and is then $m \times p$ matrix **C** $= [c_{jk}]$ with entries :

$$c_{jk} = \sum_{l=1}^{n} a_{jl} b_{lk} = a_{j1}b_{k1} + a_{j2}b_{k2} + \cdots + a_{jn}b_{nk},$$

$$j = 1, \ldots, m$$
$$k = 1, \ldots, p.$$

Matrix Multiplication by Matrix

The condition $r = n$ means that the second factor, **B**, must have as many rows as the first factor has columns, namely n. A diagram of sizes that shows when matrix multiplication is possible is as follows:

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} & = & \mathbf{C} \\ [m \times n] & [n \times p] & = & [m \times p] \end{bmatrix}$$

Matrix Multiplication

Example

$$\mathbf{AB} = \begin{bmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 22 & -2 & 43 & 42 \\ 26 & -16 & 14 & 6 \\ -9 & 4 & -37 & -28 \end{bmatrix}$$

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Here $c_{11} = 3 \cdot 2 + 5 \cdot 5 + (-1) \cdot 9 = 22$, and so on. The entry in the box is $c_{23} = 4 \cdot 3 + 0 \cdot 7 + 2 \cdot 1 = 14$. The product **BA** is not defined.

Multiplication of a Matrix and a Vector

Observe the following matrix multiplication:

$$\begin{bmatrix} 4 & 2 \\ 1 & 8 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \cdot 3 + 2 \cdot 5 \\ 1 \cdot 5 + 8 \cdot 5 \end{bmatrix} = \begin{bmatrix} 22 \\ 43 \end{bmatrix}$$

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whereas :

$$\begin{bmatrix} 3 \\ 5 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 1 & 8 \end{bmatrix}$$

is undefined.

Products of Row and Column Vectors

Example

$$\begin{bmatrix} 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = [19], \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 1 \\ 6 & 12 & 2 \\ 12 & 24 & 4 \end{bmatrix}$$

Products of Row and Column Vectors

Example

$$\begin{bmatrix} 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = [19], \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 1 \\ 6 & 12 & 2 \\ 12 & 24 & 4 \end{bmatrix}$$

CAUTION!!

Matrix Multiplication Is Not Commutative, $\mathbf{AB} \neq \mathbf{BA}$ in General.

Matrix multiplication satisfies rules similar to those for numbers, namely :

1. $(k\mathbf{A})\mathbf{B} = k(\mathbf{AB})$, *written* $k\mathbf{AB}$ *or* $\mathbf{A}k\mathbf{B}$
2. $\mathbf{A(BC)} = \mathbf{(AB)C}$, *written* $\mathbf{ABC}$
3. $(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$
4. $\mathbf{C}(\mathbf{A} + \mathbf{B}) = \mathbf{CA} + \mathbf{CB}$

Motivation of Multiplication by Linear Transformations

Let us now motivate the “unnatural” matrix multiplication by its use in linear transformations. For $n = 2$ variables these transformations are of the form

$$\begin{aligned} y_1 &= a_{11}x_1 + a_{12}x_2 \\ y_2 &= a_{21}x_1 + a_{22}x_2 \end{aligned} \tag{2.1}$$

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For instance, 2.1 may relate an x_1x_2-coordinate system to a y_1y_2-coordinate system in the plane. In vectorial form we can write 2.1 as :

Motivation of Multiplication by Linear Transformations

In vectorial form we can write 2.1 as :

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{Ax} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix} \tag{2.2}$$

Motivation of Multiplication by Linear Transformations

Now suppose further that the x_1x_2-system is related to a w_1w_2-system by another linear transformation, say,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{B}\mathbf{w} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} b_{11}w_1 + b_{12}w_2 \\ b_{21}w_1 + b_{22}w_2 \end{bmatrix} \tag{2.3}$$

Motivation of Multiplication by Linear Transformations

When the y_1y_2-system is related to the w_1w_2-system indirectly via the x_1x_2-system, and we wish to express this relation directly, substitution will show that this direct relation is a linear transformation, too, say,

$$\mathbf{y} = \mathbf{C}\mathbf{w} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} c_{11}w_1 + c_{12}w_2 \\ c_{21}w_1 + c_{22}w_2 \end{bmatrix} \tag{2.4}$$

Indeed, substituting 2.3 to 2.2, we obtain :

$$
\begin{aligned}
y_1 &= a_{11}(b_{11}w_1 + b_{12}w_2) + a_{12}(b_{21}w_1 + b_{22}w_2) \\
&= (a_{11}b_{11} + a_{12}b_{21})w_1 + (a_{11}b_{12} + a_{12}b_{22})w_2 \\
y_2 &= a_{21}(b_{11}w_1 + b_{12}w_2) + a_{22}(b_{21}w_1 + b_{22}w_2) \\
&= (a_{21}b_{11} + a_{22}b_{21})w_1 + (a_{21}b_{12} + a_{22}b_{22})w_2
\end{aligned}
$$

Comparing this with 2.4, wee see that :

$$c_{11} = a_{11}b_{11} + a_{12}b_{21}$$
$$c_{12} = a_{11}b_{12} + a_{12}b_{22}$$
$$c_{21} = a_{21}b_{11} + a_{22}b_{21}$$
$$c_{22} = a_{21}b_{12} + a_{22}b_{22}$$

This proves that $\mathbf{C} = \mathbf{AB}$

Transposition of Matrices and Vectors

We obtain the transpose of a matrix by writing its rows as columns (or equivalently its columns as rows). This also applies to the transpose of vectors. Thus, a row vector becomes a column vector and vice versa. note that, if **A** is the given matrix, then we denote its transpose by $\mathbf{A}^T$.

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Transposition of Matrices and Vectors

$$\mathbf{A} = \begin{bmatrix} 5 & -8 & 1 \\ 4 & 0 & 0 \end{bmatrix} \text{ then } \mathbf{A}^T = \begin{bmatrix} 5 & 4 \\ -8 & 0 \\ 1 & 0 \end{bmatrix}$$

Transposition of Matrices and Vectors

A little more compactly, we can write

$$\begin{bmatrix} 5 & -8 & 1 \\ 4 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 5 & 4 \\ -8 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 7 \\ 8 & -1 & 5 \\ 1 & -9 & 4 \end{bmatrix}^T = \begin{bmatrix} 3 & 8 & 1 \\ 0 & -1 & -9 \\ 7 & 5 & 4 \end{bmatrix}$$

Furthermore

$\begin{bmatrix} 6 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$. Conversely, $\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}^T = \begin{bmatrix} 6 & 2 & 3 \end{bmatrix}$

Transposition of Matrices and Vectors

Definition (Transposition of Matrices and Vectors)

The transpose of an $m \times n$ matrix $\mathbf{A} = [a_{jk}]$ is the $n \times m$ matrix (denoted as $\mathbf{A}^T$ or $\mathbf{A}'$) that has the first *row* of $\mathbf{A}$ as its first *column*, the second *row* of A as its second *column*, and so on. Thus, the transpose of $\mathbf{A}$ in Chapter (1) is written as $\mathbf{A}^T$.

$$\mathbf{A}^T = [a_{kj}] = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \ddots & \vdots \\ a_{1n} & a_{2n} & \dots & a_{nm} \end{bmatrix} \quad (3.1)$$

As a special case, transposition converts row vectors to column vectors and conversely.

Rules Transposition

Transposition gives us a choice in that we can work either with the matrix or its transpose, whichever is more convenient.
Rules for transposition are

1. $(\mathbf{A})^T = \mathbf{A}$
2. $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
3. $(c\mathbf{A})^T = c\mathbf{A}^T$
4. $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$

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CAUTION!!

Note that in (4) the transposed matrices are ***in reversed order***.

Special Matrices

Certain kinds of matrices will occur quite frequently in our work, and we now list the most important ones of them.

Definition (Symmetric and Skew-Symmetric Matrices)

Transposition gives rise to two useful classes of matrices. Symmetric matrices are square matrices whose transpose equals the matrix itself. Skew-symmetric matrices are square matrices whose transpose equals minus the matrix. Both cases are defined in 3.2 and illustrated by next example.

$$\mathbf{A}^T = \mathbf{A}, (a_{kj} = a_{jk}), \mathbf{A}^T = -\mathbf{A}, (a_{kj} = -a_{jj}, a_{jj} = 0) \tag{3.2}$$

Symmetric and Skew-Symmetric Matrices

E.g.

$$\mathbf{A} = \begin{bmatrix} 20 & 120 & 200 \\ 120 & 10 & 150 \\ 200 & 150 & 30 \end{bmatrix}$$ is symmetric, and

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$$ is skew-symmetric.

Triangular Matrices

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E.g.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}, \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{3.3}$$

Diagonal Matrices

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These are square matrices that can have nonzero entries only on the main diagonal. Any entry above or below the main diagonal must be zero.
If all the diagonal entries of a diagonal matrix **S** are equal, say, c, we call **S** a scalar matrix because multiplication of any square matrix **A** of the same size by **S** has the same effect as the multiplication by a scalar, that is,

$$\mathbf{AS} = \mathbf{SA} = c\mathbf{A} \tag{3.4}$$

Diagonal Matrices

In particular, a scalar matrix, whose entries on the main diagonal are all 1, is called a **unit matrix** (or **identity matrix**) and is denoted by $\mathbf{I}_n$ or simply by **I**. For **I**, formula 3.4 becomes

$$\mathbf{AI} = \mathbf{IA} = \mathbf{A} \tag{3.5}$$

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Diagonal Matrix D. Scalar Matrix S. Unit Matrix I

$$\mathbf{D} = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}, \mathbf{S} = \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}, \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Linear System, Coefficient Matrix, Augmented Matrix

A linear system of m equations in n unknowns $x_1, \cdots, x_n$ is a set of equations of the form

$$\begin{aligned} a_{11}x_1 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n &= b_m \end{aligned} \tag{4.1}$$

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CAUTION!!

If all the b_j are zero, then 4.1 is called a **homogeneous system**. If at least one b_j is not zero, then 4.1 is called a **nonhomogeneous system**.

Linear System, Coefficient Matrix, Augmented Matrix

Matrix Form of the Linear System 4.1. From the definition of matrix multiplication we see that the m equations of 4.1 may be written as a single vector equation

$$\mathbf{Ax} = \mathbf{b} \tag{4.2}$$

where the **coefficient matrix A** $= [a_{jk}]$ is the $m \times n$ matrix.

Geometric Interpretation. Existence and Uniqueness of Solutions

If $m = n = 2$,we have two equations in two unknowns x_1, x_2

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

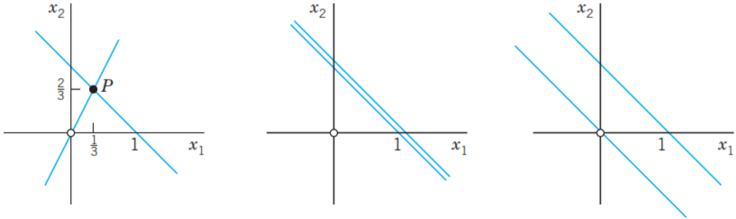
If we interpret x_1, x_2 as coordinates in the x_1x_2-plane, then each of the two equations represents a straight line, and (x_1, x_2) is a solution if and only if the point P with coordinates x_1, x_2 lies on both lines. Hence there are three possible cases (see Fig. 28 on next page):

- Precisely one solution if the lines intersect
- Infinitely many solutions if the lines coincide
- No solution if the lines are parallel

Geometric Interpretation. Existence and Uniqueness of Solutions

For instance,

Case (a)	Case (b)	Case (c)
$x_1 + x_2 = 1$	$x_1 + x_2 = 1$	$x_1 + x_2 = 1$
$2x_1 - x_2 = 0$	$2x_1 + 2x_2 = 2$	$x_1 + x_2 = 0$



Gauss Elimination

We have systems of equations:

$$
\begin{aligned}
2x_1 + 3x_2 - x_3 &= 7 \\
4x_1 + 2x_2 + 2x_3 &= 12 \\
3x_1 - x_2 + 2x_3 &= 10
\end{aligned}
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Solution steps

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\end{aligned}
$$

Solution steps

Step 1. Perform row operations to introduce 0 below a_{11}

$$
\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 4 & 2 & 2 & 12 \\ 3 & -1 & 2 & 10 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 3 & -1 & 2 & 10 \end{array}\right]
$$

Solution Steps

Step 2. Perform row operations to introduce 0 below a_{21}

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 3 & -1 & 2 & 10 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 0 & 4 & 5 & -11 \end{array}\right]$$

Solution Steps

Step 2. Perform row operations to introduce 0 below a_{21}

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 3 & -1 & 2 & 10 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 0 & 4 & 5 & -11 \end{array}\right]$$

Step 3. Perform row operations to introduce 0 below a_{31}

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 0 & 4 & 5 & -11 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Solution Steps

Step 4. Perform row operations to make a_{33} equal to 1

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & 3 & 0 & 10 \\ 0 & -4 & 0 & 10 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Solution Steps

Step 4. Perform row operations to make a_{33} equal to 1

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 0 & 10 \\ 0 & -4 & 0 & 10 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Step 5. Calculate x_3 from the last row

$x_3 = 3$

Solution Steps

Step 4. Perform row operations to make a_{33} equal to 1

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 4 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 0 & 10 \\ 0 & -4 & 0 & 10 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Step 5. Calculate x_3 from the last row

$x_3 = 3$

Step 6. Substitute x_3 back into the second row to calculate x_2

$-4x_2 = 10 \rightarrow x_2 = -\frac{10}{4} = -\frac{5}{2}$

Solution Steps

Step 7. Substitute x_3, x_2 back into the first row to calculate x_1

$$2x_1 + 3\left(-\frac{5}{2}\right) - 0 = 10$$

$$2x_1 - \frac{15}{2} = 10$$

$$2x_1 = 10 + \frac{15}{2}$$

$$2x_1 = \frac{35}{2}$$

$$x_1 = \frac{35}{4}$$

Solution

So the solution :

$$x_1 = \frac{35}{4}, x_2 = -\frac{5}{4}, x_3 = 3.$$

Problem Set

Solve the linear system given explicitly or by its augmented matrix. Show details.

- $$\begin{aligned} x + y - z &= 9 \\ 8y + 6z &= -6 \\ -2x + 4y - 6z &= 40 \end{aligned}$$

- $$\begin{bmatrix} 2 & 4 & 1 & 0 \\ -1 & 1 & -2 & 0 \\ 4 & 0 & 6 & 0 \end{bmatrix}$$

- $$\begin{aligned} 4y + 3z &= 8 \\ 2x - z &= 2 \\ 3x + 2y &= 5 \end{aligned}$$

- $$\begin{bmatrix} 4 & 1 & 0 & 4 \\ 5 & -3 & 1 & 2 \\ -9 & 2 & 1 & 5 \end{bmatrix}$$

Thank You.

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