Mathematics for Data Science

SSD23402

Chapter 5

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Example

Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3\}$ and $B = \{4, 5, 6, 7, 8\}$ then:

- $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cap B = \{4, 5\}$
- $A - B = \{1, 2, 3\}$
- $B - A = \{6, 7, 8\}$
- $\bar{A} = \{6, 7, 8, 9\}$
- $(A \bar{\cup} B) = \{9\}$

Hierarchy of Number Sets

The following provides a hierarchy of numbers, from natural numbers to complex numbers.

- **Natural** numbers: $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$
- **Prime** numbers: $\mathbf{P} = \{2, 3, 5, 7, 11, \ldots\}$
- **Composite** numbers, the set of natural numbers greater than 1 and not prime. $\mathbf{K} = \{x \mid x \in \mathbb{N}$ and $x > 1$ and $x \notin \mathbf{P}$, or $x = 4, 6, 8, 9, 10, \ldots\}$
- **Whole** numbers, natural numbers (positive integers) combined with zero. $\mathbf{C} = \mathbb{N} \cup \{0\} = \{0, 1, 2, 3, 4, \ldots\}$

Hierarchy of Number Sets

Sets and the Real Number System
Sets
The Real Number Set and Its Operations
The Real Number System
Order
Intervals
Inequalities
Absolute Value

The following provides a hierarchy of numbers, from natural numbers to complex numbers.

- The set of **integers**, denoted by $\mathbb{Z}$, is the set of negative integers combined with 0 and the set of natural numbers. In set notation:
  $\mathbb{Z} = \{..., -3, -2, -1\} \cup \mathbb{N} \cup \{0\} = \{..., -3, -2, -1, 0, 1, 2, ...\}$
- The set of fractional numbers is denoted by
  $\mathbf{P}_e = \left\{x \mid x = \frac{a}{b} \notin \mathbb{Z}\right\} = \left\{..., -\frac{1}{2}, -\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, ...\right\}$
- The set of rational numbers is denoted by $\mathbb{Q}$
  $\mathbb{Q} = \left\{x \mid x = \frac{a}{b} \text{ with } a, b \in \mathbb{Z}\right\}$
- The set of irrational numbers is denoted by $\mathbb{I}_r$
  $\mathbb{I}_r = \left\{..., \sqrt{2}, \sqrt{3}, e, \phi, ...\right\}$

The Real Number Set and Its Operations

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Example

1. Given $A \subset \mathbb{N}$ with $A = \{2, 3, 4, 5, 6, 7\}$ and $B = \{4, 5, 6, 7, 8, 9, 10\}$
   Find: $A \cup B$, $A \cap B$, $A - B$, $B - A$, $A \cap B \cup B - A$
2. Given $A \subset \mathbb{Z}$ and $B \subset \mathbb{Z}$ with
   $A = \{-5, -4, -3, -2, -1, 0, 1, 2\}$ and
   $B = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$
   Find: $A \cup B$, $A \cap B$, $A - B$, $B - A$, $A \cap B \cap B - A$.
3. Given $A \subset \mathbb{R}$, $B \subset \mathbb{R}$, and $C \subset \mathbb{R}$ with
   $A = \{x \mid -1 \leq x < 6\}$, $B = \{x \mid 3 \leq x \leq 9\}$, and
   $C = \{x \mid 10 \leq x < 12\}$
   Find: $A \cup B$, $A \cap B$, $(A \cup B) \cap C$, $(A \cap B) \cup C$, $A - B$

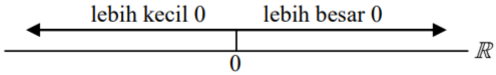
Properties of the Real Number System

For $x, y, z \in \mathbb{R}$, the following properties hold:

1. Commutative Property:
   - $x + y = y + x$
   - $x \cdot y = y \cdot x$
2. Associative Property:
   - $x + (y + z) = (x + y) + z$
   - $x(yz) = (xy)z$
3. Distributive Property:
   $x(y + z) = xy + xz$
4. Identity Elements:
   - 0 is the additive identity such that $x + 0 = x$ (addition)
   - 1 is the multiplicative identity such that $x \cdot 1 = x$ (multiplication)
5. Inverse Elements:
   - For every $x \in \mathbb{R}$, there exists an additive inverse $(-x)$ such that $x + (-x) = 0$
   - For every $x \neq 0$, there exists a multiplicative inverse x^{-1}

Order

The set of real numbers is divided by 0 into two parts, the part greater than 0, called **positive** numbers, and the part smaller than 0, called **negative** numbers.



Properties of Order

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In the real number system, the properties of order hold as stated in the following properties.

1. Trichotomy: For any two numbers x and y, exactly one of the following holds: $x < y$, $x = y$, or $x > y$.
2. Transitive: If $x < y$ and $y < z$, then $x < z$.
3. Addition: $x < y \iff x + z < y + z$ for any z.
4. Multiplication:
   - If $z > 0$, then $x < y \iff xz < yz$.
   - If $z < 0$, then $x < y \iff xz > yz$.

Intervals

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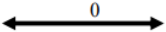
An interval is another way to write subsets of real numbers, denoted as $(..., ...)$, $(..., ...]$, $[..., ...)$, or $[..., ...]$. The notation “(“ and “)” represent open intervals, and “[" and “]” represent closed intervals.

Intervals

Sets and the Real Number System
- Sets
- The Real Number Set and Its Operations
- The Real Number System
- Order
- Intervals
- Inequalities
- Absolute Value

No	Selang dan gambarnya		Himpunan
1	(a,b)		$x \mid a<x<b$
2	$(a,b]$		$x \mid a<x\leq b$
3	$[a,b)$		$x \mid a\leq x<b$
4	$[a,b]$		$x \mid a\leq x\leq b$
5	$(-\infty,b]$		$x \mid x\leq b$
6	$(-\infty,b)$		$x \mid x<b$
7	$[a,\infty)$		$x \mid x\geq a$
8	(a,∞)		$x \mid x>a$
9	$(-\infty,\infty)$		R

Example

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Draw and write the set notation for the following intervals:

1. $(-2, 5)$
2. $(-\infty, 1]$
3. $[-1, \infty)$
4. $(-5, -3]$
5. $[1, 4]$

Inequalities

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Example

Find the value of x that satisfies the inequality $2x - 4 < 0$

$$
\begin{aligned}
2x - 4 &< 0 \\
2x - 4 + 4 &< 0 + 4 \\
2x &< 4 \\
x &< 2
\end{aligned}
$$

Absolute Value

Definition (Absolute Value)

The absolute value of a, written as $|a|$, is defined as:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Properties of Absolute Value

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The following are the properties of absolute value:

a.	$\pm a \le \lvert a \rvert$	b.	$\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$
c.	$\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a \rvert}{\lvert b \rvert}$, $b \ne 0$	d.	$\lvert a+b \rvert \le \lvert a \rvert + \lvert b \rvert$
e.	$\lvert a-b \rvert \le \lvert a-b \rvert$		

The following shows inequalities involving absolute value:

a.	$\lvert x \rvert < a \Leftrightarrow -a < x < a$	
b.	$\lvert x \rvert > a \Leftrightarrow x < -a \cup x > a$	
c.	$\lvert x \rvert \le a \Leftrightarrow -a \le x \le a$	
d.	$\lvert x \rvert \ge a \Leftrightarrow x \le -a \cup x \ge a$	

Thank You.

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