Mathematics for Data Science

SSD23402

Chapter 6

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Definition of Relations

Definition (Relation)

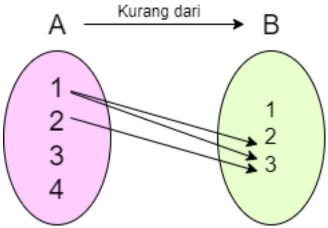
A relation between two sets A and B is a pairing of the elements of set A with elements of set B.

Examples of relations in mathematics include: greater than, less than, half of, factor of, and so on.

Example

Given $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$. If the relation from set A to set B is defined as "less than," consider the following diagram:

The "Less than" Relation



The diagram above is called an arrow diagram. The direction of the relation is shown by arrows, and the name of the relation is "less than."

Definition of Functions

Definition (Function)

A relation from set A to set B is called a function or mapping if and only if each element in set A is paired with exactly one element in set B.

Function Notation

Suppose f is a function from set A to set B. The function f is denoted as:

$$f : A \to B \tag{2.1}$$

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Definition of Functions

- Set A is called the domain or the set of definition, denoted as D_f.
- Set B is called the codomain or the co-domain of function f, denoted as K_f.
- The set of all elements in B that have a pairing in A is called the *range* or image, denoted as R_f.

Example

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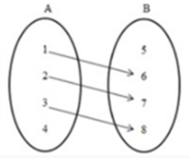
<tr><th>Contoh 1</th><th>Contoh 2</th><th>Contoh 3</th></tr>
<tr>
<td>
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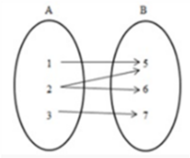
<td>
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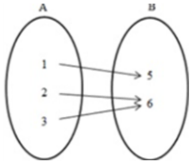
<td>
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<tr>
<td>Bukan fungsi karena terdapat anggota di A yang tidak dihubungkan dengan anggota di B</td>
<td>Bukan fungsi karena terdapat anggota di A yang dihubungkan lebih dari satu dengan anggota di B</td>
<td>Merupakan fungsi karena setiap anggota di A tepat dihubungkan dengan satu anggota di B</td>
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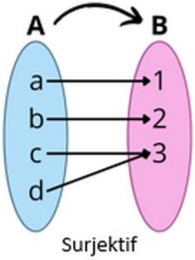
Types of Functions

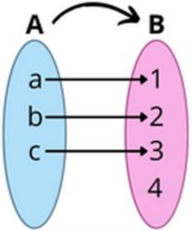
Surjective Function

If every element in set B has a pairing with an element in set A, then f is called a surjective function or an onto function.



Injective Function

If each element in set B has a unique pairing in A, then f is called an injective function or a one-to-one function.



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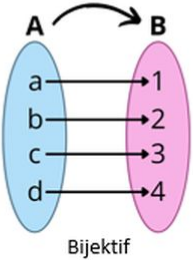
Bijective Function

If every element in set B has exactly one pairing in A, then f is called a bijective function or a one-to-one correspondence. It can be understood that a one-to-one correspondence is both a surjective and injective function.



Operations on Functions

Given a real scalar a and functions f and g, the sum $f+g$, difference $f-g$, scalar multiplication $a \cdot f$, product $f \cdot g$, and quotient $\frac{f}{g}$ are defined as follows:

1. $(f+g)(x) = f(x) + g(x)$
2. $(f-g)(x) = f(x) - g(x)$
3. $(a \cdot f)(x) = a \cdot f(x)$
4. $(f \cdot g)(x) = f(x)g(x)$
5. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Example

Example

Given:

$$f(x) = 2x - 4$$
$$g(x) = -3x + 2$$

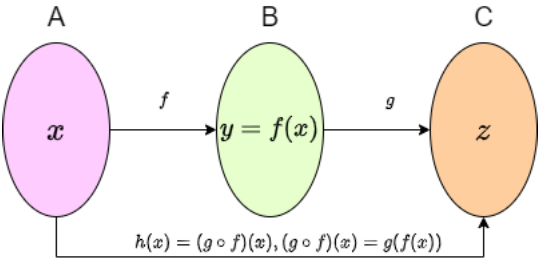
Find:

1. $f + g$
2. $f - g$
3. $f \cdot g$
4. $\frac{f}{g}$

Composite Functions

Definition

A composite function is a function involving more than one function. When one function is followed by another function, a new function is formed. This new function is the composite of the two previous functions.



Properties of Composite Functions

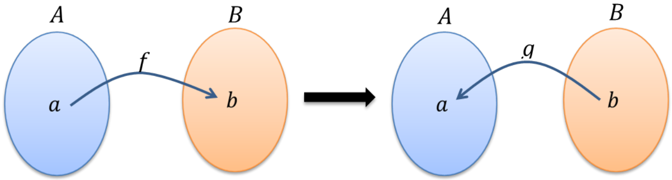
Properties

1. Non-Commutative: $(f \circ g)(x) \neq (g \circ f)(x)$
2. Associative: $(f \circ (g \circ h))(x) = ((f \circ g) \circ h)(x)$
3. Has an identity function: $(f \circ I)(x) = (I \circ f)(x) = f(x)$

Inverse Functions

Definition

Given a function $f : A \to B$, the inverse (reverse) of a function is a relation g from B to A.



Inverse Functions

- Generally, the inverse of a function is not necessarily a function.
- If $f : A \to B$ is a one-to-one correspondence, then the inverse of f is also a function.
- The notation for an inverse function is f^{-1}.

Forms of Inverse Functions

Function	Inverse
$f(x) = ax + b$	$f^{-1}(x) = \frac{y-b}{a}$
$f(x) = x^n$	$f^{-1}(x) = \sqrt[n]{y}$
$f(x) = \frac{ax+b}{cx+d}$	$f^{-1}(x) = \frac{-dx+b}{cx-a}$
$f(x) = ax^2 + bx + c$	$f^{-1}(x) = \frac{-b+\sqrt{b^2-4ac}}{2a}$
$f(x) = \log_a(cx)$	$f^{-1}(x) = \frac{a^x}{c}$
$f(x) = a^{cx}$	$f^{-1}(x) = \log_a\left(x^{\frac{1}{c}}\right)$

Thank You.

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