Mathematics for Data Science

SSD23402

Chapter 8

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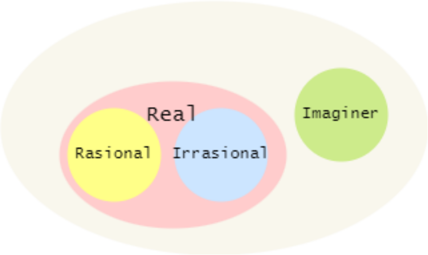
Understanding Complex Numbers

A Complex Number is a combination of a real number and an imaginary number.



Understanding Imaginary Numbers

Imaginary Numbers

An imaginary number is defined as the square root of -1 and is commonly represented by i. In mathematics, i is the imaginary unit that allows us to work with complex numbers. Imaginary numbers do not have a true real value and only exist in the context of complex numbers.

In General

A number that is the square root of a negative number.

Example:

1. $\sqrt{-5}$
2. $\sqrt{-7}$

Understanding Complex Numbers

Complex Numbers

A complex number z is denoted as a pair of real numbers (x, y) and can be written as $z = (x, y)$. The value x is the real part of z, and y is the imaginary part of z, denoted as $x = \text{Re}(z)$ and $y = \text{Im}(z)$.

General Form of Complex Numbers

Notation

A complex number z can be represented in the following general form:

$$z = a + bi$$

In this form:

1. a is the real part of the complex number.
2. b is the imaginary part of the complex number.
3. i is the imaginary unit, defined as $\sqrt{-1}$.

General Form of Complex Numbers

Examples of complex numbers in general form

1. $3 + 2i$ has a real part $a = 3$ and an imaginary part $b = 2$.
2. $-4 - 7i$ has a real part $a = -4$ and an imaginary part $b = -7$.

This general form allows us to clearly distinguish between the real and imaginary parts of a complex number and is used for performing various arithmetic operations on complex numbers.

Properties of the Field of Complex Numbers

The set of all complex numbers $\mathbb{C}$, along with the operations of addition and multiplication, forms a **field**.

1. $z_1 + z_2 \in \mathbb{C}$ and $z_1 z_2 \in \mathbb{C}$, $\forall z_1, z_2 \in \mathbb{C}$
2. $z_1 + z_2 = z_2 + z_1$ and $z_1 z_2 = z_2 z_1$, $\forall z_1, z_2 \in \mathbb{C}$
3. $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$ and $(z_1 z_2) z_3 = z_1 (z_2 z_3)$, $\forall z_1, z_2, z_3 \in \mathbb{C}$
4. $z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$, $\forall z_1, z_2, z_3 \in \mathbb{C}$
5. There exists $0 = (0, 0) \in \mathbb{C}$, such that $z + 0 = z$
6. There exists $1 = (1, 0) \neq 0, 1 \in \mathbb{C}$, such that $z \cdot 1 = 1 \cdot z = z$
7. For every $z = (x, y) \in \mathbb{C}$, there exists $-z = (-x, -y) \in \mathbb{C}$ such that $z + (-z) = (-z) + z = 0$
8. For every $z = (x, y) \in \mathbb{C}$, $z \neq 0$, there exists $z^{-1} = \left(\frac{x}{x^2+y^2}, \frac{-y}{x^2+y^2} \right) \in \mathbb{C}$ such that $z z^{-1} = z^{-1} z = 1$

Exercise 1

Given two complex numbers $z = 2 + 3i$ and $w = 1 - 2i$. Calculate $z + w$ and $z \cdot w$.

Solution

Addition:

$$z + w = (2 + 3i) + (1 - 2i) = 3 + i$$

Multiplication:

$$z \cdot w = (2 + 3i) \cdot (1 - 2i) = 8 - i$$

Exercise 2

Calculate z^2 and w^3, with $z = 1 + 2i$ and $w = 3 - 4i$.

Solution

Square of z^2:

$$z^2 = (1 + 2i)^2 = 1 + 4i - 4 = -3 + 4i$$

Cube of w^3:

$$w^3 = (3 - 4i)^3 = (3 - 4i)(3 - 4i)(3 - 4i) = -117 - 44i$$

Cartesian Notation of Complex Numbers

A complex number z in Cartesian notation is expressed as:

$$z = a + bi$$

Where:

- a is the real part of z
- b is the imaginary part of z
- i is the imaginary unit, with $i^2 = -1$

With Cartesian notation, you can perform algebraic operations such as addition, subtraction, multiplication, and division on complex numbers.

Example of addition in Cartesian notation

$$z_1 = a_1 + b_1 i, \quad z_2 = a_2 + b_2 i$$

$$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i$$

Polar Notation of Complex Numbers

Polar notation is used to represent complex numbers in terms of modulus and argument.
A complex number $z = a + bi$ can be expressed in polar notation as:

$$z = r \cdot (\cos(\theta) + i \sin(\theta))$$

Where:

- r is the modulus of z, denoted as $|z|$
- θ is the argument of z, denoted as $\arg(z)$

Relationship between Cartesian and Polar notation:

$$r = |z| = \sqrt{a^2 + b^2}$$

$$\theta = \arg(z) = \arctan\left(\frac{b}{a}\right)$$

Converting Cartesian to Polar Notation

To convert Cartesian notation $z = a + bi$ to polar notation, we use the following formulas:

$$|z| = \sqrt{a^2 + b^2} \quad \text{and} \quad \arg(z) = \arctan\left(\frac{b}{a}\right)$$

Thus, the complex number z in polar notation is:

$$z = |z| \cdot (\cos(\arg(z)) + i\sin(\arg(z)))$$

Converting Polar to Cartesian Notation

To convert polar notation $z = |z| \cdot (\cos(\theta) + i\sin(\theta))$ to Cartesian notation, we use the following formulas:

$$a = |z| \cdot \cos(\theta) \quad \text{and} \quad b = |z| \cdot \sin(\theta)$$

Therefore, the complex number z in Cartesian notation is:

$$z = a + bi$$

Thank You.

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