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Mathematics for Data Science

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Chapter 10

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Complex Numbers in Geometry

Points and Vectors

Geometric Representation of Complex Numbers

Addition of Complex Numbers

Rotation and Translation

Polynomial Equations and Complex Roots

Polynomial Equations

Fundamental Theorem of Algebra

Complex Roots

Applications in Engineering and Science

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Definition of a Point in the Complex Plane

A complex number $z = a + bi$ can be interpreted as the point (a, b) in the Cartesian plane. The real part a represents the x -coordinate, and the imaginary part b represents the y -coordinate.

Definition of a Vector in the Complex Plane

A complex number $z = a + bi$ can also be interpreted as a vector originating from the point $(0, 0)$ to the point (a, b) in the Cartesian plane.



Geometric Representation of Complex Numbers

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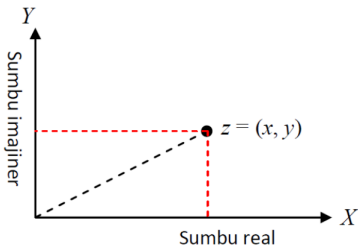
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Geometrically, complex numbers can be represented in a plane. This plane is called the complex plane, referring to Cartesian coordinates, which consist of the real axis (X-axis) and the imaginary axis (Y-axis).



Modulus of the Sum of Two Complex Numbers

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Recall the definition of the modulus of a complex number from the previous discussion. For any complex number $z = x + iy$, the modulus is given by $|z| = \sqrt{x^2 + y^2}$. If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, then:

$$|z_2 - z_1| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$





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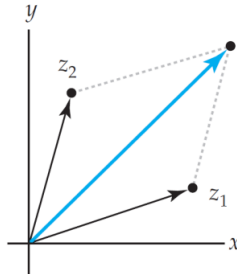
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Geometrically, there is a one-to-one correspondence between the sum of two complex numbers $z_1 + z_2$ and the diagonal of a parallelogram formed by z_1 and z_2 , as shown in the following figure.



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Rotation with Complex Numbers

To rotate a vector z by an angle θ counterclockwise, we can multiply it by $e^{i\theta}$, where θ is the angle of rotation.

$$z' = ze^{i\theta}$$

Translation with Complex Numbers

To translate a vector z by (a, b) , we can add $a + bi$ to it.

$$z' = z + (a + bi)$$



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Example 1

Rotation of the vector $z = 3 + 4i$ by 90° counterclockwise.

$$z' = (3 + 4i)e^{i\frac{\pi}{2}}$$



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Example 2

Translation of the vector $z = 2 + 2i$ by $(1, -1)$.

$$z' = (2 + 2i) + (1 - 1i)$$





Definition of Polynomial Equations

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- 1 A polynomial equation is a mathematical equation that contains terms of various degrees of a variable.
- 2 The roots of the equation are the values of the variable that satisfy the equation.
- 3 Sometimes, the roots of an equation may be complex numbers.





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Polynomial Equations

Definition

A polynomial equation is a mathematical equation consisting of several terms with the highest non-zero degree of the variable.

Example

$$ax^2 + bx + c = 0$$

This is an example of a quadratic polynomial equation, where a , b , and c are coefficients and x is the variable.



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Fundamental Theorem of Algebra

Fundamental Theorem of Algebra

Every polynomial equation with complex coefficients has at least one complex root.

This is one of the most fundamental theorems in mathematics, showing that complex roots exist in various polynomial equations.

Finding the Roots of Polynomial Equations

- 1 Some equations can be solved by factorization.
- 2 Quadratic equations can be solved using the quadratic formula.
- 3 Equations with complex roots can be solved using complex numbers.



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Definition

A complex number is a number that has both a real part and an imaginary part. The roots of complex numbers are the solutions of equations whose roots are complex numbers.

Example

$$x^2 + 1 = 0$$

This equation has no real roots, but it has complex roots $x = \pm i$, where i is the imaginary unit.



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Example

$$x^2 + 2x + 1 = 0$$

Solution: $x = -1$

$$x^2 + 4 = 0$$

Solution: $x = \pm 2i$





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AC Circuit Analysis

Polynomial equations are used in AC circuit analysis to understand the behavior of current and voltage in the circuit.

Fourier Transform

The Fourier Transform is a mathematical tool used to analyze signals and waves in various applications, including communications and signal processing.



Thank You.

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