DISCRETE MATHEMATICS
Meeting 5

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1. Real Number System
    - Real Number Set and Its Operations
    - Real Number System
    - Order
    - Intervals
    - Inequalities
    - Absolute Value

Hierarchy of Number Sets

Here is the hierarchy of numbers, starting from natural numbers to complex numbers.

- **Natural** numbers: $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$
- **Prime** numbers: $\mathbf{P} = \{2, 3, 5, 7, 11, \ldots\}$
- **Composite** numbers, the set of natural numbers greater than 1 and not prime numbers.
  $\mathbf{K} = \{x \,|\, x \in \mathbb{N} \text{ and } x > 1 \text{ and } x \notin \mathbf{P}, \text{ or } x = 4, 6, 8, 9, 10, \ldots\}$
- **Whole** numbers, natural numbers (positive integers) combined with the number zero. $\mathbf{C} = \mathbb{N} \cup \{0\} = \{0, 1, 2, 3, 4, \ldots\}$

Here is the hierarchy of numbers, starting from natural numbers to complex numbers.

- **Integer** set, denoted by $\mathbb{Z}$, includes negative integers combined (added) with zero and natural numbers. Expressed in set notation: $\mathbb{Z} = \{..., -3, -2, -1\} \cup \mathbb{N} \cup \{0\} = \{..., -3, -2, -1, 0, 1, 2, ...\}$
- The set of fractions is denoted as $\mathbf{P}_e = \{x \mid x = \frac{a}{b} \notin \mathbb{Z}\} = \left\{..., -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, ...\right\}$
- The set of rational numbers is denoted by $\mathbb{Q}$ $\mathbb{Q} = \{x \mid x = \frac{a}{b} \text{where} a, b \in \mathbb{Z}\}$
- The set of irrational numbers is denoted by $\mathbb{I}_r$ $\mathbb{I}_r = \left\{..., \sqrt{2}, \sqrt{3}, e, \phi, ...\right\}$

Real Number Set and Its Operations

Example

1. Given $A \subset \mathbb{N}$ with $A = \{2, 3, 4, 5, 6, 7\}$ and $B = \{4, 5, 6, 7, 8, 9, 10\}$
   Determine: $A \cup B$, $A \cap B$, $A - B$, $B - A$, $A \cap B \cup B - A$
2. Given $A \subset \mathbb{Z}$ and $B \subset \mathbb{Z}$ with $A = \{-5, -4, -3, -2, -1, 0, 1, 2\}$ and $B = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$
   Determine: $A \cup B$, $A \cap B$, $A - B$, $B - A$, $A \cap B \cap B - A$.
3. Given $A \subset \mathbb{R}$, $B \subset \mathbb{R}$, and $C \subset \mathbb{R}$ with $A = \{x \mid -1 \leq x < 6\}$ and $B = \{x \mid 3, x \leq 9\}$ and $C = \{x \mid 10 \leq x < 12\}$
   Determine: $A \cup B$, $A \cap B$, $(A \cup B) \cap C$, $(A \cap B) \cup C$, $A - B$

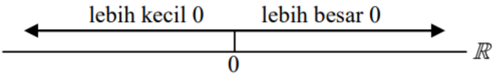
Properties of the Real Number Field

Untuk $x, y, z \in \mathbb{R}$ maka

1. sifat komutatif:
   a. $x + y = y + x$
   b. $x.y = y.x$
2. sifat asosiatif:
   a. $x + (y + z) = (x + y) + z$
   b. $x(yz) = (xy)z$
3. sifat distributif: $x(y + z) = xy + xz$
4. unsur satuan (identitas):
   a. 0 sehingga $x + 0 = x$ (penambahan)
   b. 1 sehingga $x.1 = x$ (perkalian)
5. unsur invers (balikan):
   a. $\forall x \in \mathbb{R}$ ada invers $(-x)$ sehingga $x + (-x) = 0$
   b. $\forall x \neq 0$ ada invers x^{-1} sehingga $x.x^{-1} = 1$.

Order

The set of real numbers is separated by the number 0 into two parts: the part greater than 0 is called **positive** numbers and the part smaller than 0 is called **negative** numbers.



Properties of Order

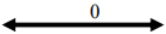
The following properties of order apply in the real number system as shown in these properties.

1.	Trikhotomi	:	untuk setiap dua bilangan x dan y hanya berlaku salah satu dari hubungan, $x < y$ atau $x = y$ atau $x > y$.
2.	Transitif	:	jika $x < y$ dan $y < z$ maka $x < z$.
3.	Penambahan	:	$x < y \Leftrightarrow x + z < y + z$.
4.	Pengalian	:	a. jika $z > 0$ maka $x < y \Leftrightarrow xz < yz$ b. jika $z < 0$ maka $x < y \Leftrightarrow xz > yz$.

Intervals

Intervals are an alternative way to write subsets of real numbers, denoted as $(..., ...), (..., ...], [..., ...)$, or $[..., ...]$. The notation "(" and ")" denotes open intervals, and "[" and "]" denotes closed intervals.

Intervals

No	Selang dan gambarnya		Himpunan
1	(a,b)		$x \mid a < x < b$
2	$(a,b]$		$x \mid a < x \leq b$
3	$[a,b)$		$x \mid a \leq x < b$
4	$[a,b]$		$x \mid a \leq x \leq b$
5	$(-\infty,b]$		$x \mid x \leq b$
6	$(-\infty,b)$		$x \mid x < b$
7	$[a,\infty)$		$x \mid x \geq a$
8	(a,∞)		$x \mid x > a$
9	$(-\infty,\infty)$		R

Example

Draw the interval and write the set notation for:

1. $(-2, 5)$
2. $(-\infty, 1]$
3. $[-1, \infty)$
4. $(-5, -3]$
5. $[1, 4]$

Inequalities

Example

Find the value of x that satisfies the inequality $2x - 4 < 0$

$$\begin{aligned} 2x - 4 &< 0 \\ 2x - 4 + 4 &< 0 + 4 \\ 2x &< 4 \\ x &< 2 \end{aligned}$$

Absolute Value

Definition (Absolute Value)

The absolute value of a, denoted $|a|$, is defined as:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Properties of Absolute Value

The following are the properties of absolute value:

a. $\pm a \leq |a|$

b. $|ab| = |a||b|$

c. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, $b \neq 0$

d. $|a+b| \leq |a| + |b|$

e. $|a-b| \leq |a-b|$

The following is an inequality in absolute value:

a.	$\lvert x \rvert < a \Leftrightarrow -a < x < a$	
b.	$\lvert x \rvert > a \Leftrightarrow x < -a \cup x > a$	
c.	$\lvert x \rvert \leq a \Leftrightarrow -a \leq x \leq a$	
d.	$\lvert x \rvert \geq a \Leftrightarrow x \leq -a \cup x \geq a$