Mathematics for Data Science

SSD23402

Chapter 11

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Daftar Isi

Definition of Limits

Introduction to the concept of limits

The concept of limits is one of the essential concepts in mathematics used to understand the behavior of functions as they approach a specific value.

Limit notation: $\lim_{x \to a} f(x) = L$

This notation is used to express the limit of the function $f(x)$ as x approaches the value a, meaning $f(x)$ will approach L.

Graphical Approach

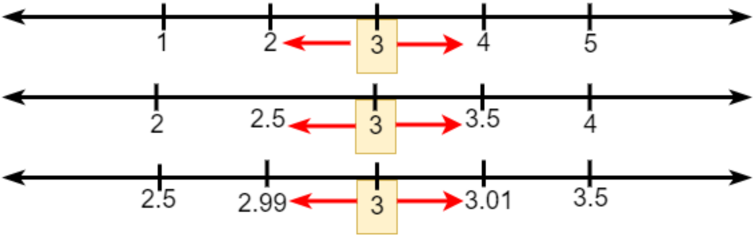
Using function graphs

Using function graphs to illustrate the concept of limits helps in understanding how the function approaches a particular value.

Concept of approaching a specific value

In this concept, we observe how the graph of the function approaches a particular value as the variable x approaches a certain point.

Concept of Approaching a Specific Value



It turns out that there are many values approaching 3, both from the left and right. If we denote these numbers that approach 3 as x, then x is said to be approaching 3.

Properties of Limits

Properties of Limits

1. $\lim_{x\to a}(mx+b) = ma+b$
2. $\lim_{x\to a} k = k$
3. $\lim_{x\to a} x = a$
4. $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$
5. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ for $\lim_{x\to a} f(x) > 0$ and even n.

Properties of Limits

Properties of Limit Operations

1. Addition of limits:
$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. Subtraction of limits:
$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. Multiplication of limits:
$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
4. Division of limits: $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, provided $\lim_{x \to a} g(x) \neq 0$

Example

Calculate the following limit:

$$\lim_{x\to 2}(3x^2 - 5x + 7)$$

Using the properties of addition and multiplication of limits, we can calculate the limit as follows:

$$\lim_{x\to 2}(3x^2 - 5x + 7) = \lim_{x\to 2} 3x^2 - \lim_{x\to 2} 5x + \lim_{x\to 2} 7$$

$$= 3\lim_{x\to 2} x^2 - 5\lim_{x\to 2} x + 7\lim_{x\to 2} 1$$

$$= 3(2^2) - 5(2) + 7(1) = 12 - 10 + 7 = 9$$

Therefore, $\lim_{x\to 2}(3x^2 - 5x + 7) = 9$.

Example

We can calculate the limit as follows:

$$\lim_{x\to 3}(2x^2 - 3x + 1) = 2\lim_{x\to 3} x^2 - 3\lim_{x\to 3} x + \lim_{x\to 3} 1$$

$$= 2(3^2) - 3(3) + 1 = 18 - 9 + 1 = 10$$

Therefore, $\lim_{x\to 3}(2x^2 - 3x + 1) = 10$.

Example

Calculate the following limit:

$$\lim_{x \to 0} \frac{x^2 - 3x}{x - 2}$$

We can use the properties of division of limits:

$$\lim_{x \to 0} \frac{x^2 - 3x}{x - 2} = \frac{\lim_{x \to 0}(x^2 - 3x)}{\lim_{x \to 0}(x - 2)}$$

$$= \frac{(0^2 - 3 \cdot 0)}{(0 - 2)} = \frac{0}{-2} = 0$$

Therefore, $\lim_{x \to 0} \frac{x^2 - 3x}{x - 2} = 0$.

Introduction

Definition

One-sided limits refer to approaching from the left side or the right side.

1. x approaches a from the **left**, written as $x \to a^-$
2. x approaches a from the **right**, written as $x \to a^+$

Introduction

Definition

1. $f(x) \to L$ as $x \to a^-$, called the **left-hand limit** of $f(x)$ as $x \to a^-$, written as $\lim_{x\to a^-} f(x) = L$
2. $f(x) \to L$ as $x \to a^+$, called the **right-hand limit** of $f(x)$ as $x \to a^+$, written as $\lim_{x\to a^+} f(x) = M$

Example

We will solve the left-hand limit of the function $f(x) = x^2 + 1$ as x approaches 1. The left-hand limit is expressed as:

$$\lim_{x \to 1^-} (x^2 + 1)$$

We will calculate this limit.

Solution

To calculate the limit, we need to approach $x = 1$ from smaller values. Let's calculate:

$$\lim_{x \to 1^-} (x^2 + 1) = \lim_{x \to 1^-} (x^2) + \lim_{x \to 1^-} (1)$$

Now, we can evaluate each limit.

$$\lim_{x \to 1^-} (x^2) = 1^2 = 1$$

$$\lim_{x \to 1^-} (1) = 1$$

Thus, we have calculated the left-hand limit of $f(x) = x^2 + 1$ as $x \to 1$, and the result is:

$$\lim_{x \to 1^-} (x^2 + 1) = 1 + 1 = 2$$

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Right-Hand Limit of $f(x) = x^2 + 1$ as $x \to 1$

We want to calculate the right-hand limit of the function $f(x) = x^2 + 1$ as x approaches 1^+, meaning from the right side of $x = 1$.

$$\lim_{x \to 1^+} (x^2 + 1)$$

To calculate the right-hand limit, we can directly substitute $x = 1$ into the function $f(x) = x^2 + 1$:

$$\lim_{x \to 1^+} (x^2 + 1) = (1^2 + 1) = 2$$

Therefore,

$$\lim_{x \to 1^+} (x^2 + 1) = 2$$

Limits at Infinity

Types of Limits at Infinity

1. x getting larger towards ∞; and
2. x getting smaller towards $-\infty$.

Example:

1. $\lim_{x\to\infty} \frac{1}{x} = 0$
2. $\lim_{x\to\infty} \frac{1}{x^2} = 0$
3. $\lim_{x\to-\infty} \frac{1}{x} = 0$
4. $\lim_{x\to-\infty} \frac{1}{x^3} = 0$

Properties of Limits at Infinity

In mathematics, there are important properties of limits at infinity:

1. $\lim_{x\to\infty} c = c$ and $\lim_{x\to-\infty} c = c$
2. $\lim_{x\to\infty}[f(x) + g(x)] = \lim_{x\to\infty} f(x) + \lim_{x\to\infty} g(x)$
3. $\lim_{x\to\infty}[f(x) \cdot g(x)] = \lim_{x\to\infty} f(x) \cdot \lim_{x\to\infty} g(x)$
4. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{\lim_{x\to\infty} f(x)}{\lim_{x\to\infty} g(x)}$
5. $\lim_{x\to\infty}[f(g(x))] = \lim_{x\to\infty} f(\lim_{x\to\infty} g(x))$

Problem

Calculate the following limit:

$$\lim_{x\to\infty} \frac{4x+1}{2x-3}$$

To calculate the limit as $x \to \infty$, we can use the ratio of the highest degree coefficients in the numerator and denominator:

$$\lim_{x\to\infty} \frac{4x+1}{2x-3} = \frac{\text{Highest degree coefficients in numerator}}{\text{Highest degree coefficients in denominator}}$$

$$= \frac{4}{2} = 2$$

$$\lim_{x\to\infty} \frac{4x+1}{2x-3} = 2$$

Infinite Limits

Definition

An infinite limit refers to a limit with an **infinite value**. There are two types: (a) $\lim_{x\to a} f(x) = \infty$ and (b) $\lim_{x\to a} f(x) = -\infty$

Example

1. $\lim_{x\to 0^+} \frac{1}{x} = \infty$
2. $\lim_{x\to 0^-} \frac{x}{x+2} = -\infty$
3. $\lim_{x\to 2^+} \frac{1}{x} = -\infty$
4. $\lim_{x\to 2^-} \frac{1}{x} = \infty$

Thank You.

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