Mathematics for Data Science

SSD23402

Chapter 12

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Using dividing factors

One of the methods for calculating limits is by using dividing factors to simplify complex limit forms.

Eliminating indeterminate forms

We can eliminate indeterminate forms such as $\frac{0}{0}$ or $\frac{\infty}{\infty}$ using certain methods to obtain accurate limit results.

Simplification Method Using Dividing Factors

Introduction

This method is used to simplify undefined or complex limit forms. A dividing factor is an expression that contains the conjugate form of the observed function.

Steps

1. Identify the indeterminate form in the limit.
2. Calculate the dividing factor using the conjugate form.
3. Multiply both the numerator and the denominator by the dividing factor.
4. Simplify the expression.
5. Calculate the limit of the simplified expression.

Example Problem

Calculate the following limit:

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

Solution:

$$\begin{aligned}
\lim_{x \to 2} \frac{x^2 - 4}{x - 2} &= \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} \\
&= \lim_{x \to 2} (x+2) \quad \text{(Eliminating the dividing factor)} \\
&= 2 + 2 = 4 \quad \text{(Calculating the limit)}
\end{aligned}$$

Introduction

Limit Simplification Method

This method is used to calculate the limit of a function by eliminating indeterminate forms, such as $\frac{0}{0}$ or $\frac{\infty}{\infty}$, so that we can obtain accurate limit results.

Problem

Calculate the following limit:

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

Example Problem

Solution

We can use the simplification method by eliminating the indeterminate form:

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(x + 2)}{x - 2}$$

Now, we can simplify the equation by canceling $x - 2$ in the denominator:

$$= \lim_{x \to 2} (x + 2)$$

Finally, we substitute $x = 2$:

$$= 2 + 2 = 4$$

Thus, $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4$.

Basic Limit Theorems

Comparison Theorem

The Comparison Theorem is used to compare the limit of a function with a simpler function to determine the limit result.

Comparison Theorem in Limits

Comparison Theorem

If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$ with $L, M \in \mathbb{R}$, then:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$$

provided $M \neq 0$.

The Comparison Theorem allows us to calculate the limit of a complex function by comparing it to a simpler function.

Example Problem

Calculate the following limit:

$\lim_{x\to 5}(3x-11) = 4$ and $\lim_{x\to 5}(x+3) = 8$

thus:

Using the Comparison Theorem:

- $\lim_{x\to 5}\left[(3x-11)\cdot(x+3)\right] =$
  $\lim_{x\to 5}(3x-11)\cdot\lim_{x\to 5}(x+3) = 4\cdot 8 = 32$
- $\lim_{x\to 5}\frac{(3x-11)}{(x+3)} = \frac{4}{8} = \frac{1}{2}$
- $\lim_{x\to 5}\left[3x-11\right]^3 = 4^3 = 64$
- $\lim_{x\to 5}\sqrt[5]{x+3} = \sqrt[3]{8} = 2$

Limits of Algebraic Functions

Definition

A limit of an algebraic function is the limit of a polynomial function as the variable approaches a specific value.

Example

For example, we want to calculate:

$$\lim_{x \to 2} (3x^2 - 2x + 1)$$

Example Calculation of Limits of Polynomial Functions

Suppose we want to calculate:

$$\lim_{x \to 2} (3x^2 - 2x + 1)$$

- We can directly substitute x with 2:

$$\lim_{x \to 2} (3x^2 - 2x + 1) = 3(2^2) - 2(2) + 1 = 12 - 4 + 1 = 9$$

- Therefore, $\lim_{x \to 2} (3x^2 - 2x + 1) = 9$.

Thank You.

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