DISCRETE MATHEMATICS
Meeting 6

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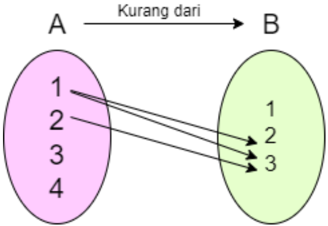
Definition of Relations

Definition

A relation between two sets A and B is the pairing of elements in set A with elements in set B.

Examples of relations in mathematics include: greater than, less than, half of, a factor of, etc. Example: Given $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$, if we define a relation from set A to set B as "less than," refer to the following diagram:

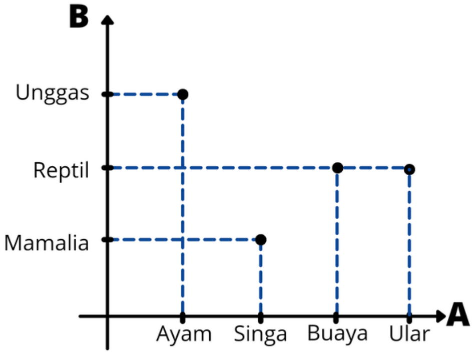
Mapping Relations



The diagram above is called an arrow diagram. The direction of the relation is shown by arrows, and the name of the relation is "less than."

Coordinate Relations

A relation in coordinate form is a way to represent relationships between elements in a set using pairs of values (coordinates). Examples include sets of points in two- or three-dimensional space.



Matrix Relations

We can represent this relation in matrix form using the domain as rows and the codomain as columns. This matrix is known as the relation matrix.

Suppose we have the following relation between elements in two sets: A and B.

$$R = \{(a, x), (b, y), (c, z)\} \tag{1}$$

We can illustrate this relation using a relation matrix.

Table: Relation Matrix

	x	y	z
a	1	0	0
b	0	1	0
c	0	0	1

Here, the value 1 indicates a relationship exists between the respective row and column elements, while 0 indicates no relationship.

Definition of Functions

Definition (Function)

A relation from set A to set B is called a function or mapping if and only if each element in set A pairs with exactly one element in set B.

Function Notation

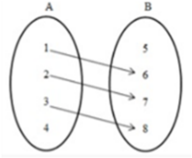
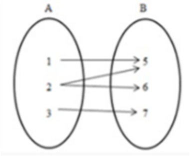
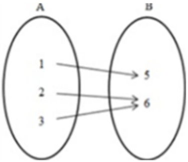
Suppose f is a function from set A to set B, then the function f is represented as:

$$f : A \to B \tag{2}$$

Definition of Functions

- Set A is called the domain, denoted D_f.
- Set B is called the codomain or target set of function f, denoted K_f.
- The set of all elements in B that have a pair in A is called the *range*, denoted R_f.

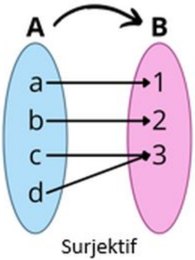
Example

Contoh 1	Contoh 2	Contoh 3
		
Bukan fungsi karena terdapat anggota di A yang tidak dihubungkan dengan anggota di B	Bukan fungsi karena terdapat anggota di A yang dihubungkan lebih dari satu dengan anggota di B	Merupakan fungsi karena setiap anggota di A tepat dihubungkan dengan satu anggota di B

Types of Functions

Surjective Function

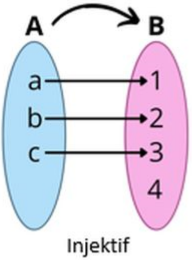
If every element in set B has a partner in set A, then f is called a surjective function or onto function.



Types of Functions

Injective Function

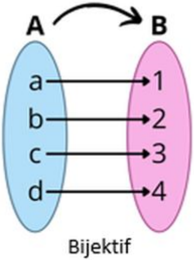
If each element in set B has a unique partner in set A, then f is called an injective function or one-to-one function.



Types of Functions

Bijective Function

If each element in set B has exactly one partner in set A, then f is called a bijective function or one-to-one correspondence. It is both surjective and injective.



Function Operations

Given a real scalar a and functions f and g. The sum $f+g$, difference $f-g$, scalar product $a \cdot f$, product $f \cdot g$, and quotient $\frac{f}{g}$ are defined as follows:

1. $(f+g)(x) = f(x) + g(x)$
2. $(f-g)(x) = f(x) - g(x)$
3. $(a \cdot f)(x) = a \cdot f(x)$
4. $(f \cdot g)(x) = f(x)g(x)$
5. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Example

Example

Given:

$$f(x) = 2x - 4$$
$$g(x) = -3x + 2$$

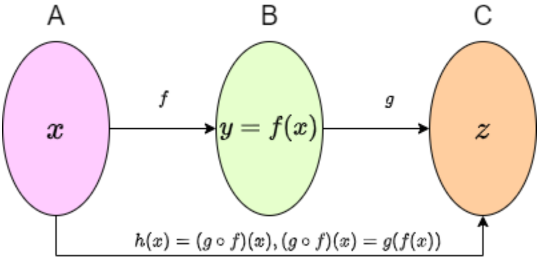
Find:

1. $f + g$
2. $f - g$
3. $f \cdot g$
4. $\frac{f}{g}$

Composition of Functions

Definition

The composition of functions involves combining more than one function. When a function is followed by another function, it forms a new function. This new function is the composition result of the two previous functions.



Properties of Function Composition

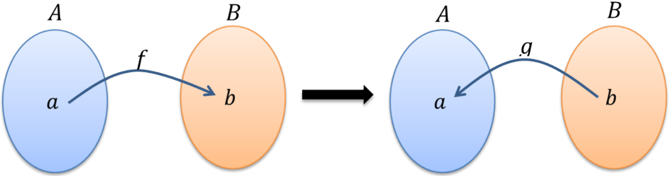
Properties

1. Non-Commutative, $(f \circ g)(x) \neq (g \circ f)(x)$
2. Associative, $(f \circ (g \circ h))(x) = ((f \circ g) \circ h)(x)$
3. Has an identity function, $(f \circ I)(x) = (I \circ f)(x) = f(x)$

Inverse Functions

Definition

Given a function $f : A \to B$. The inverse of a function is a relation g from B to A.



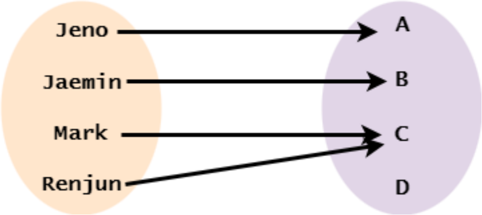
Inverse Functions

- Generally, the inverse of a function may not necessarily be a function.
- If $f : A \to B$ is a one-to-one correspondence, then the inverse of function f is also a function.
- The notation for an inverse function is f^{-1}.

Form of Inverse Functions

Function	**Inverse**
$f(x) = ax + b$	$f^{-1}(x) = \frac{y-b}{a}$
$f(x) = x^n$	$f^{-1}(x) = \sqrt[n]{y}$
$f(x) = \frac{ax+b}{cx+d}$	$f^{-1}(x) = \frac{-dx+b}{cx-a}$
$f(x) = ax^2 + bx + c$	$f^{-1}(x) = \frac{-b+\sqrt{b^2-aa(c-x)}}{2a}$
$f(x) = \log_a cx$	$f^{-1}(x) = \frac{a^x}{c}$
$f(x) = a^{cx}$	$f^{-1}(x) = \log_a x^{\frac{1}{c}}$

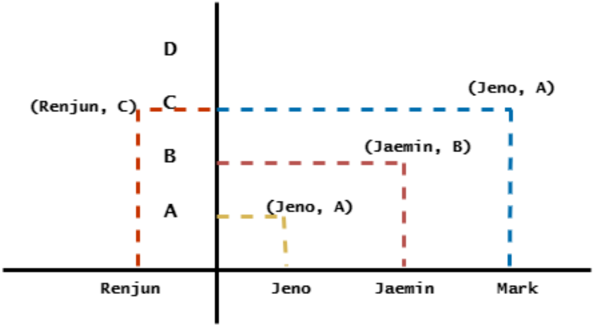
Before discussing function graphs conceptually, let's review the illustration of function $f : K \to N$ as shown below:



$D_f = \{\text{Jeno, Jaemin, Mark, Renjun}\}$ and $K_f = \{A < B, C, D\}$. The values of function $f(\text{Jeno}) = A, f(\text{Jaemin}) = B, f(\text{Mark}) = C, f(\text{Renjun}) = C$ and $R_f = \{A, B, C\}$.

Graphing Functions

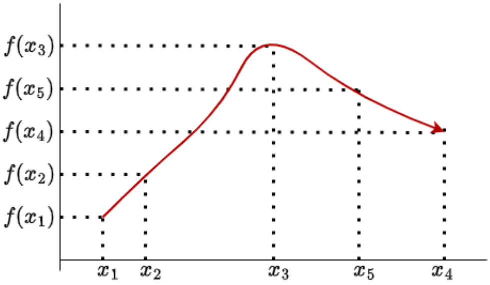
Then, we create a coordinate system with K as the horizontal axis and N as the vertical axis. Ordered pairs between points on K and N form a set of points. This set of points represents the graph of function $f : K \to N$ as follows:



Definition

Suppose $y = f(x)$ is a function. The graph of function f is defined as the set of ordered pairs of points $x, f(x) \mid x \in D(f)$.

A rough sketch of the graph can be obtained by determining several ordered pairs $x, f(x)$ on the xy-coordinate system, then connecting these points as in the following figure:



Graphing Functions

Draw the graph of the function $y = f(x) = 2x$.

Graph of Inverse Functions

There are two ways to graph an inverse function. The first is by directly graphing the inverse function, and the second is by reflecting the function over the line $y = x$.

Example

Draw the inverse function of $y = 2x + 6$

Graph of Inverse Functions