Composite and Inverse Functions

Composite Functions

Composite Functions

The **composite function** $(f \circ g)(x)$ is defined as

$$(f \circ g)(x) = f\lceil g(x) \rceil$$

When we are given f(x) and g(x), to find $(f \circ g)(x)$ we substitute g(x) for x in f(x) to get $f\lceil g(x) \rceil$.

Composite Functions

Example Given $f(x) = x^2 - 2x + 3$ and $g(x) = x - 5$, find

a) $f(4)$

To find f(4), we substitute 4 for each x in f(x).

$$f(x) = x^2 - 2x + 3$$

$$f(4) = 4^2 - 2 \cdot 4 + 3 = 16 - 8 + 3 = 11$$

b) $f(a)$

To find *f(a)*, we substitute a for each *x* in *f(x)*.

$$f(x) = x^2 - 2x + 3$$

$$f(a) = a^2 - 2a + 3$$

One-to-One Functions

One-to-One Function

A function is **one-to-one** if each element in the range corresponds to exactly one element in the domain.

Consider the following two set of ordered pairs.

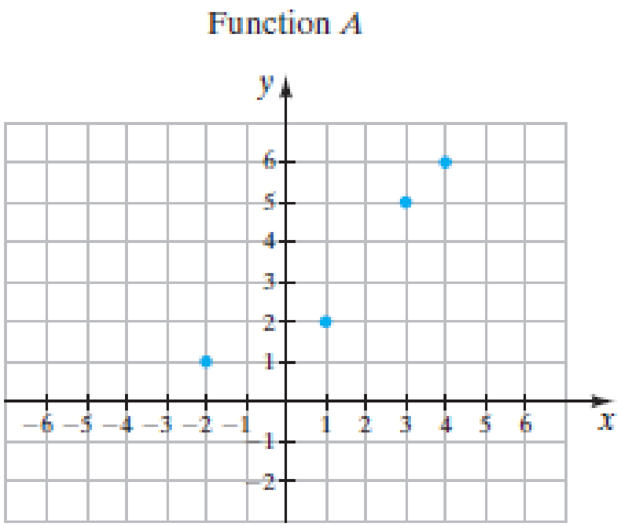
A = {(1,2), (3,5), (4,6), (-2,1)}

B = {(1,2), (3,5), (4,6), (-2,5)}

- Both sets A and B represent functions since each x-coordinate corresponds to exactly one y-coordinate.
- Set A is also a one-to-one function since each y-coordinate also corresponds to exactly one x-coordinate.
- Set B is not a one-to-one function since the y-coordinate 5 corresponds to two x-coordinates, 3 and -2.

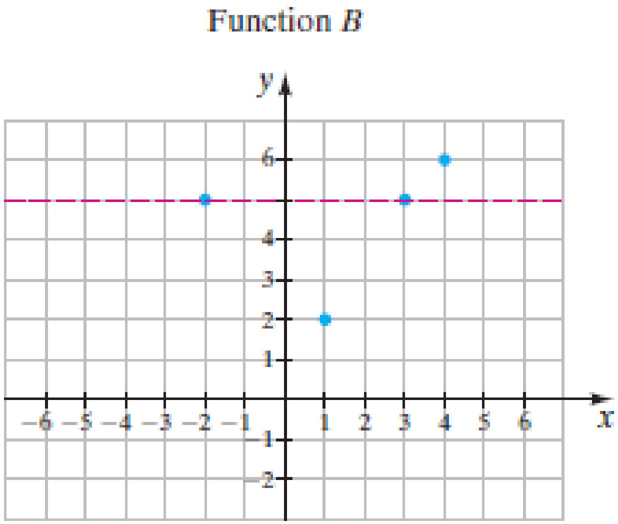
One-to-One Functions

The graphs below show functions A and B.



- Each x-coordinate corresponds to exactly one y-coordinate.

  Therefore, set A is a function.
- Each y-coordinate also corresponds to exactly one x-coordinate. *Therefore, set A is a one-to-one function.*



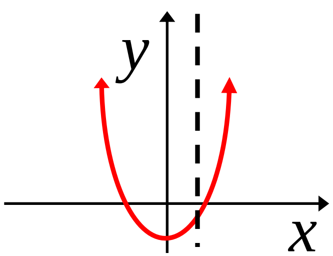
- Each x-coordinate corresponds to exactly one y-coordinate.

  Therefore, set B is a function.
- The y-coordinate 5 corresponds to two x-coordinates, 3 and −2. *Therefore, set B is not a one-to-one function.*

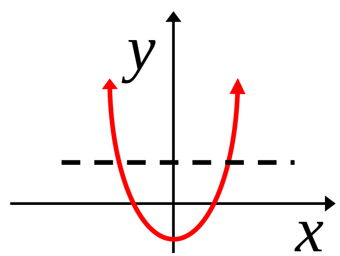
One-to-One Functions

Horizontal Line Test

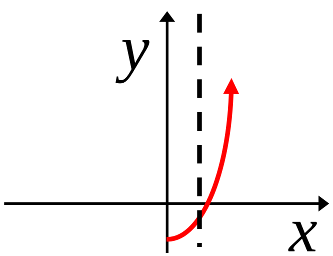
If a horizontal line can be drawn so that it intersects the graph of a function at more than one point, the graph is not a one-to-one function.



Function



Not a one-to-one function



One-to one function

Find Inverse Functions

Inverse Function

If f(x) is a one-to-one function with ordered pairs of the form (x, y), the **inverse function**, $f^{-1}(x)$, is a one-to-one function with ordered pairs of the form (y, x).

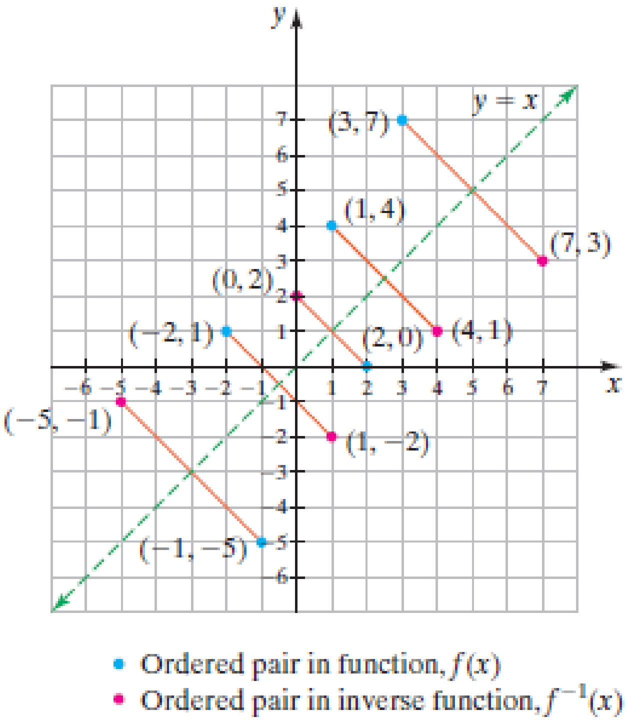
Example

Function, f(x):	{(1,4), (2,0), (3,7), (-2,1), (-1,-5)}
Inverse function, $f^{-1}(x)$	{(4,1), (0,2), (7,3), (1,-2), (-5,-1)}

continued

Find Inverse Functions

If we graph the points in the function and the points in the inverse function we see that the points are symmetric with respect to the line y = x.



Find Inverse Functions

Some important information about inverse functions:

- Only one-to-one functions have inverse functions.
- The domain of $f(x)$ is the range of $f^{-1}(x)$.
- The range of $f(x)$ is the domain of $f^{-1}(x)$.
- When the graph of $f(x)$ and $f^{-1}(x)$ are graphed on the same axes, the graphs of $f(x)$ and $f^{-1}(x)$ are symmetric about the line $y = x$.

Find Inverse Functions

To Find the Inverse Function of a One-to-One Function

1. Replace f(x) with y.
2. Interchange the two variables x and y.
3. Solve the equation for y.
4. Replace y with $f^{-1}(x)$ (this gives the inverse function using inverse function notation).

Find Inverse Functions

Example Find the inverse function of f(x) = 4x + 2, and graph both f(x) and $f^{-1}(x)$.

$$f(x) = 4x + 2$$

$$y = 4x + 2$$ **Replace f(x) with y.**

$$x = 4y + 2$$ **Interchange x and y.**

$$x - 2 = 4y$$

$$\frac{x-2}{4} = y$$

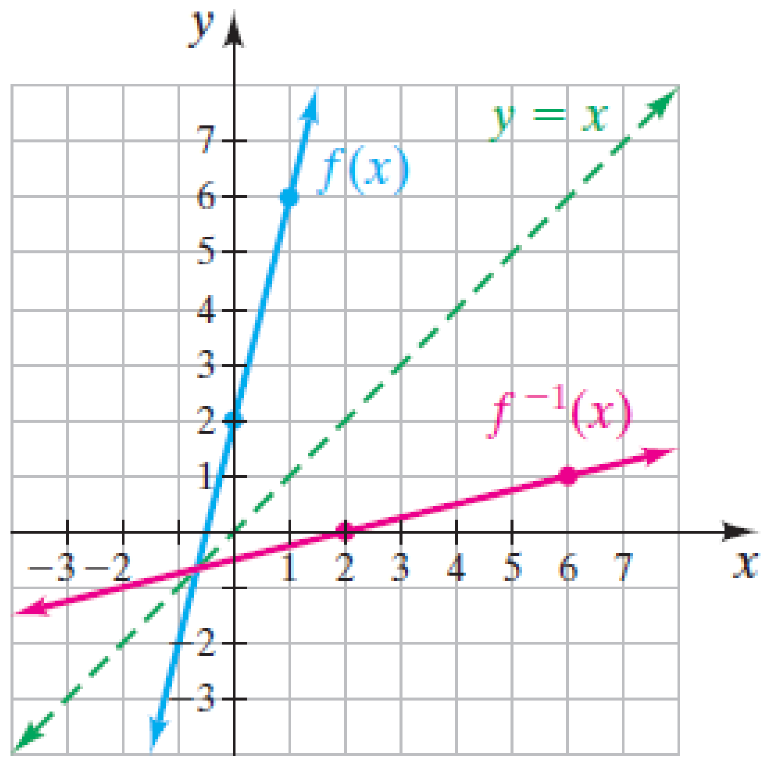
$$f^{-1}(x) = \frac{x-2}{4}$$

continued

Find Inverse Functions

x	$y = f(x)$
0	2
1	6

x	$y = f^{-1}(x)$
2	0
6	1



Composition of a Function and Its Inverse

The Composition of a Function and Its Inverse
For any one-to-one function f(x) and its inverse $f^{-1}(x)$,
$(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$

Example If $f(x) = 4x + 2$, $f^{-1}(x) = \dfrac{x-2}{4}$, show that $(f \circ f^{-1})(x) = x$

continued

Composition of a Function and Its Inverse

To determine $(f \circ f^{-1})(x)$, substitute $f^{-1}(x)$ for each x in f(x).

$$f(x) = 4x + 2$$

$$(f \circ f^{-1})(x) = 4\left(\frac{x-2}{4}\right) + 2$$

$$= x - 2 + 2 = x$$