MATEMATIKA DISKRIT

Meeting 9

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Introduction

Matrices, which are rectangular arrays of numbers or functions, and vectors are the main tools of linear algebra. Matrices are important because they let us express large amounts of data and functions in an organized and concise form. Furthermore, since matrices are single objects, we denote them by single letters and calculate with them directly. All these features have made matrices and vectors very popular for expressing scientific and mathematical ideas.

Matrix

A **matrix** is a rectangular array of numbers or functions which we will enclose in brackets. For example,

$$\begin{bmatrix} 0.3 & 1 & -5 \\ 0 & -0.2 & 16 \end{bmatrix}, \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \tag{1}$$

$$\begin{bmatrix} e^{-2} & 2x^2 \\ e^{6x} & 4x \end{bmatrix}, \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}, \begin{bmatrix} 4 \\ \frac{1}{2} \end{bmatrix} \tag{2}$$

are matrices.

Linear Systems, a Major Application of Matrices

We are given a system of linear equations, briefly a **linear system**, such as :

$$4x_1 + 6x_2 + 9x_3 = 6$$
$$6x_1 - 2x_3 = 20$$
$$5x_1 - 8x_2 + x_3 = 10$$

where x_1, x_2, x_3 are the **unknowns**. We form the **coefficient matrix**, call it **A**, by listing the coefficients of the unknowns in the position in which they appear in the linear equations. In the second equation, there is no unknown x_2 which means that the coefficient of is 0 and hence in matrix **A**, $a_{22} = 0$, Thus :

$$\mathbf{A} = \begin{bmatrix} 4 & 6 & 9 \\ 6 & 0 & -2 \\ 5 & -8 & 1 \end{bmatrix}$$

We form another matrix $\begin{bmatrix} 4 & 6 & 9 & 6 \\ 6 & 0 & -2 & 20 \\ 5 & -8 & 1 & 10 \end{bmatrix}$ by augmenting **A** with the right sides of the linear system and call it the augmented matrix of the system.

General Concepts and Notations

Let us formalize what we just have discussed. We shall denote matrices by capital boldface letter **A,B,C,** . . . , or by writing the general entry in brackets; thus $\mathbf{A} = [a_{ij}]$, and so on.

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$$\mathbf{A} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \tag{3}$$

vectors

A vector is a matrix with only one row or column. Its entries are called the components of the vector. We shall denote vectors by lowercase boldface letters **a**,**b**, . . . or by its general component in brackets,$\mathbf{a} = [a_{ij}]$, and so on. Our special vectors in (1) suggest that a (general) **row vector** is of the form

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$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}.$$

For Instance, $\mathbf{a} = \begin{bmatrix} -2 & 5 & 0.8 & 0 & 1 \end{bmatrix}$

Vectors

A **column vector** is of the form ;

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \text{ For Instance : } \mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 7 \\ -4 \end{bmatrix}$$

Addition and Scalar Multiplication of Matrices and Vectors

Definition (Equality of Matrices)

Two matrices $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$ are equal, written $\mathbf{A} = \mathbf{B}$, if and only if they have the same size and the corresponding entries are equal, that is, $a_{11} = b_{11}, a_{12} = b_{12}$ and so on. Matrices that are not equal are called **different**. Thus, matrices of different sizes are always different.

Addition and Scalar Multiplication of Matrices and Vectors

Definition (Addition of Matrices)

The **sum** of two matrices $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$ ***of the same size*** is written $\mathbf{A}+\mathbf{B}$ and has entries $a_{ij} + b_{ij}$ tained by adding the corresponding entries of $\mathbf{A}$ and $\mathbf{B}$. Matrices of different sizes cannot be added.

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As a special case, the **sum** $\mathbf{a}+\mathbf{b}$ of two row vectors or two column vectors, which must have the same number of components, is obtained by adding the corresponding components.

Addition and Scalar Multiplication of Matrices and Vectors

Definition (Scalar Multiplication (Multiplication by a Number))

The **product** of any $m \times n$ matrix $\mathbf{A} = [a_{ij}]$ and any **scalar** c (number c) is written $c\mathbf{A}$ and is the $m \times n$ matrix $c\mathbf{A} = [ca_{ij}]$ obtained by multiplying each entry of $\mathbf{A}$.

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Here $(-1)\mathbf{A}$ is simply written $-\mathbf{A}$ and is called the **negative** of $\mathbf{A}$. Similarly, $(-k)\mathbf{A}$ is written $-k\mathbf{A}$. Also $\mathbf{A} + (-\mathbf{B})$ is written **A-B** and is called the difference of $\mathbf{A}$ and $\mathbf{B}$ (which must have the same size!).

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Here $\mathbf{0}$ denotes the **zero matrix**(of size $m \times n$), that is the $m \times n$ matrix with all entries zero. If $m = 1$ or $n = 1$, this is a vector, called a **zero vector**

Rules for Matrix Addition and Scalar Multiplication

Hence matrix addition is ***commutative*** and ***associative***. Similarly, for scalar multiplication we obtain the rules :

1. $c(\mathbf{A}+\mathbf{B}) = c\mathbf{A} + c\mathbf{B}$
2. $(c+k)+\mathbf{A} = c\mathbf{A} + k\mathbf{A}$
3. $c(k\mathbf{A}) = (ck)\mathbf{A}$
4. $1\mathbf{A} = \mathbf{A}$

Problem Set!

$$\text{Let}: \mathbf{A} = \begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 5 & 2 \\ 5 & 3 & 4 \\ -2 & 4 & -2 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 0 & 3 \\ 3 & 4 \\ 3 & -1 \end{bmatrix},$$

$$\mathbf{u} = \begin{bmatrix} 1.5 \\ 0 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} -5 \\ -30 \\ 10 \end{bmatrix}$$

Problem Set

Find the following expressions, indicating which of the rules before, give reasons why they are not defined.

1. $2\mathbf{A} + 4\mathbf{B}, 4\mathbf{B} + 2\mathbf{A}$
2. $4(3\mathbf{A}), 2(5\mathbf{D} + 4\mathbf{C})$
3. $(\mathbf{C} + \mathbf{D}) + \mathbf{E}, (\mathbf{D} + \mathbf{E}) + \mathbf{C}$
4. $(\mathbf{u}+\mathbf{v}) - \mathbf{w}$