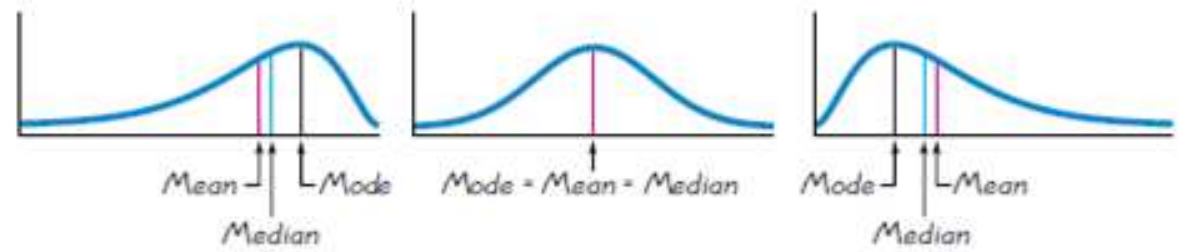




CENTER SIZE AND LOCATION

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Introduction



- One of the most important aspects of describing data distribution is the central tendency.
- Any arithmetic measurement aimed at describing a value that represents the center or central value of a cluster of data (set of observations) is known as a measure of data centering (central tendency).

- There are three measures of data centralization that are often used, namely:
 1. Mean (arithmetic average)
 2. Median
 3. Mode

Other statistical measures are Geometric Mean, Harmonic Mean (H) as well as some important characteristics to understand for a good measure of central tendency and how to choose or use the right central tendency value.

- Measures of data location: Quartiles, Deciles, percentiles

Mean Count 1

Single Data

Suppose $X_1, X_2, X_3, \dots, X_n$ are the observations from the sample, then the calculated average of the data set is

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \dots + X_n}{n} = \frac{\sum_{i=1}^n X_i}{n}$$

▶ Example

If the statistics exam scores of 5 students from a class are 70, 75, 60, 65, and 80. Then the mean is

$$\bar{X} = \frac{70 + 75 + 60 + 65 + 80}{5} = \frac{350}{5} = 70$$

Mean Count 2

Repeated data

Suppose the repeated data values with a certain frequency, X_1 repeated f_1 , X_2 repeated f_2 , X_3 repeated f_3 , ..., X_n repeated f_n are the observations of the sample, then the calculated average of the data set is

$$\bar{X} = \frac{f_1 X_1 + f_2 X_2 + f_3 X_3 + \dots + f_n X_n}{f_1 + f_2 + f_3 + \dots + f_n} = \frac{\sum_{i=1}^n f_i X_i}{\sum_{i=1}^n f_i}$$

Example

If in a statistics exam, 3 students got 65, 3 students got 70, 5 students got 80, and 2 students got 100. Then the calculated average value:

$$\bar{X} = \frac{(3 \times 65) + (3 \times 70) + (5 \times 80) + (2 \times 100)}{3 + 3 + 5 + 2} = \frac{1005}{13} = 77,31$$

Mean Count 3

Weighted data

Suppose a data where each data has a certain weight, X1 value with weight B1, X2 value with weight B2, X3 value with weight B3, ..., and Xn value with weight Bn, then the calculated average value is:

$$\bar{X} = \frac{B_1 X_1 + B_2 X_2 + B_3 X_3 + \dots + B_n X_n}{B_1 + B_2 + B_3 + \dots + B_n} = \frac{\sum_{i=1}^n B_i X_i}{\sum_{i=1}^n B_i}$$

Example

The way to calculate the final grade of a course is

$$NA = 10\% \text{ absensi} + 20\% \text{ tugas} + 30\% \text{ UTS} + 40\% \text{ UAS}$$

A student who always attends class, his average assignment is 80, his UTS is 70, and his UAS is 75, then the final grade for the student is:

$$NA = \frac{(10\% \times 100) + (20\% \times 80) + (30\% \times 70) + (40\% \times 75)}{10\% + 20\% + 30\% + 40\%} = \frac{77}{100\%} = 77$$

Mean Count 3

Group Data

$$\bar{X} = \frac{\sum_{i=1}^n f_i X_i}{\sum_{i=1}^n f_i}$$

Or

$$\bar{X} = X_0 + p \left(\frac{\sum_{i=1}^n f_i c_i}{\sum_{i=1}^n f_i} \right)$$

where:

- f_i = frequency of the i-th class
- X_i = midpoint of the i-th class

where:

- X_0 = midpoint of the class with code 0
- f_i = frequency of the i-th class
- c_i = code of the i-th class
- p = class width

Example

Class Interval	f	Mid Value	$f_i X_i$
31 - 40	2	35.5	71
41 - 50	3	45.5	136.5
51 - 60	5	55.5	277.5
61 - 70	14	65.5	917
71 - 80	24	75.5	1812
81 - 90	20	85.5	1710
91 - 100	12	95.5	1146
Total	80		6070

$$\begin{aligned}\bar{X} &= \frac{\sum_{i=1}^n f_i X_i}{\sum_{i=1}^n f_i} \\ &= \frac{6070}{80} \\ &= 75.875\end{aligned}$$

Example

Class Interval	f_i	Mid Value	c_i	$f_i c_i$
31 - 40	2	35.5	-4	-8
41 - 50	3	45.5	-3	-9
51 - 60	5	55.5	-2	-10
61 - 70	14	65.5	-1	-14
71 - 80	24	75.5	0	0
81 - 90	20	85.5	+1	+20
91 - 100	12	95.5	+2	+24
Total	80			3

$$\begin{aligned}\bar{X} &= X_0 + p \left(\frac{\sum_{i=1}^n f_i c_i}{\sum_{i=1}^n f_i} \right) \\ &= 75.5 + 10 \left(\frac{3}{80} \right) \\ &= 75.875\end{aligned}$$

Mean Measure 1

Single data

Suppose $X_1, X_2, X_3, \dots, X_n$ are observations from the sample, then the average measure (U) of the data set is:

$$U = \sqrt[n]{X_1 \times X_2 \times X_3 \times \dots \times X_n}$$

But if the observation result is too large then:

$$\log U = \frac{\sum_{i=1}^n \log X_i}{n}$$

Example

Calculate the average of the numbers 25, 102, 354, and 1610!

$$U = \sqrt[4]{25 \times 102 \times 354 \times 1610} = 195,25$$

Atau

$$\log U = \frac{\log 25 + \log 102 + \log 354 + \log 1610}{4} = \frac{9,16}{4} = 2.290$$

$$U = 10^{2.290} = 195,25$$

Mean Measure 2

Group Data

$$\log U = \frac{\sum_{i=1}^n (f_i \log x_i)}{\sum_{i=1}^n f_i}$$

where:

- f_i = frequency of the i -th class
- x_i = midpoint of the i -th class

Example

Class Interval	f_i	Mid Value	$\log x_i$	$f_i \log x_i$
31 - 40	2	35.5	1.55	3.10
41 - 50	3	45.5	1.66	4.97
51 - 60	5	55.5	1.74	8.72
61 - 70	14	65.5	1.82	25.43
71 - 80	24	75.5	1.88	45.07
81 - 90	20	85.5	1.93	38.64
91 - 100	12	95.5	1.98	23.76
Total	80			149.69

$$\log U = \frac{\sum_{i=1}^n (f_i \log x_i)}{\sum_{i=1}^n f_i}$$
$$= \frac{149.69}{80} = 1.87 \leftrightarrow U = 74,33$$

Harmonic mean

Single data

Suppose $X_1, X_2, X_3, \dots, X_n$ are observations from a sample, then the harmonic mean (H) of the data set is

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

Example

Calculate the harmonic mean for the data set: 3, 5, 6, 6, 7, 10, 12!

$$H = \frac{7}{\frac{1}{3} + \frac{1}{5} + \frac{1}{6} + \frac{1}{6} + \frac{1}{7} + \frac{1}{10} + \frac{1}{12}}$$
$$= 5 \frac{145}{167}$$

Harmonic mean

$$H = \frac{\sum_{i=1}^n f_i}{\sum_{i=1}^n \left(\frac{f_i}{x_i} \right)}$$

where:

- f_i = frequency of the i -th class
- x_i = midpoint of the i -th class

Example

Class Interval	f_i	Mid value	$\frac{f_i}{x_i}$
31 - 40	2	35.5	0.06
41 - 50	3	45.5	0.07
51 - 60	5	55.5	0.09
61 - 70	14	65.5	0.21
71 - 80	24	75.5	0.32
81 - 90	20	85.5	0.23
91 - 100	12	95.5	0.13
Total			1.10

$$\begin{aligned} H &= \frac{\sum_{i=1}^n f_i}{\sum_{i=1}^n \left(\frac{f_i}{x_i} \right)} \\ &= \frac{80}{1.10} \\ &= 72.49 \end{aligned}$$

Excercise

The following data represents the survival to death, measured to the nearest tenth of a minute, of a random sample of 60 flies that have been sprayed with a new chemical in a laboratory experiment.

2.4	1.6	3.2	4.6	0.4	1.8	2.7	1.7	5.3	1.2
0.7	2.9	3.5	0.9	2.1	2.4	0.4	3.9	6.3	2.5
3.9	2.6	1.8	3.4	2.3	1.3	2.8	1.1	0.2	2.1
2.8	3.7	3.1	1.5	2.3	2.6	3.5	5.9	2.0	1.2
2.8	3.7	3.1	1.5	2.3	2.6	3.5	5.9	2.0	1.2
1.3	2.1	0.3	2.5	4.3	1.8	1.4	2.0	1.9	1.7

1. Determine the calculated average of both single data and group data
2. Determine the measured average
3. Determine the harmonic mean