Harmonic Mean and Mode

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Harmonic Mean

Single data

Suppose X1, X2, X3, ..., Xn are observations from a sample, then the harmonic mean (H) of the data set is

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

Harmonic Mean Formula

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

where H is the harmonic mean, n is the number of data points, and x_i is the i-th data value.

Example

A bridge match consists of 10 tables. In this match, the average length of time played in a set of bridge cards is desired. In the first match, the playing time for each set of cards at each table was calculated.
The results are as follows (in minutes).
7, 6, 8, 10, 8, 8, 9, 12, 9, 11

What is the harmonic mean length of the match?

Answer:

Given $n = 10$, using the harmonic mean formula:

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

$$= \frac{10}{\frac{1}{7} + \frac{1}{6} + \cdots + \frac{1}{11}}$$

$$= 8.467$$

Example

Calculate the harmonic mean for the data 4, 5, 4, 40, 3, 5, 6, 5.

Answer:

Given $n = 8$, then:

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

$$= \frac{8}{\frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{5}}$$

$$= 4.92$$

Harmonic Mean

Group Data

$$H = \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n} \left(\frac{f_i}{x_i} \right)}$$

dengan :

x_i = nilai tengah kelas ke-i

f_i = frekuensi kelas ke-i

Example

Class Interval	f	Mid Value	Fi/xi
31 – 40	2	35.5	0.06
41 – 50	3	45.5	0.07
51 – 60	5	55.5	0.09
61 – 70	14	65.5	0.21
71 – 80	24	75.5	0.32
81 – 90	20	85.5	0.23
91 - 100	12	95.5	0.13
Total	80		1.10

$$H = \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n} \left(\frac{f_i}{x_i} \right)}$$

$$= \frac{80}{1.10}$$

$$= 72.49$$

Mode

A mode is an explanation of a group of data by using the value that appears frequently in the data group. Or it can also be said to be a popular (fashionable) value in a group of data.

If a data group has more than one data value that appears frequently, then the data set has more than one mode. A group of data that has two modes is called bimodal, while if more than two modes are called multimodal.

If in a group of data there is not a single data value that appears frequently, then the group of data is considered to have no mode.

Mode

Ungrouped Data

Example: 2, 8, 9, 11, 2, 6, 6, 7, 5, 2 → therefore, **Mode = 2**

Ten students were selected as a sample and their heights were measured.

The results of the height measurements are as follows:

172, 167, 180, 170, 169, 160, 175, 165, 173, 170

Determine the mode of the students' heights!

To make it easier to observe and find the mode, we can also sort the data. The sorted data is as follows:

160, 165, 167, 169, **170, 170**, 172, 173, 175, 180

It is easy to determine that the mode is **170**.

Mode

Formula for Mode of Grouped Data

$$Mo = b + \left(\frac{b_1}{b_1 + b_2} \right) p$$

Explanation:

- Mo = Mode
- b = lower boundary of the class interval with the highest frequency
- p = width of the class interval
- $b_1 = f_m - f_{m-1}$ (frequency of the modal class minus the frequency of the class before it)
- $b_2 = f_m - f_{m+1}$ (frequency of the modal class minus the frequency of the class after it)

Example

Class Interval	f
31 – 40	2
41 – 50	3
51 – 60	5
61 – 70	14
71 – 80	24
81 – 90	20
91 - 100	12
Total	80

Based on the table above, it is obtained:

$b = 70{,}5$

$p = 10$

$b_1 = 10$

$b_2 = 4$

$$Mo = 70{,}5 + 10\left(\frac{10}{10+4}\right) = 77{,}64$$

Example

Nilai	Frekuensi
31 – 40	3
41 – 50	5
51 – 60	10
61 – 70	11
71 – 80	8
81 – 90	3

Take another look at the grouped data table!

The **mode** of the data in the grouped data table above is...

From the grouped data table, it can be determined that the mode lies between **61 – 70** because this class has the **highest frequency**, which is **11**.

Based on the table beside, we obtain:

- $b = 60.5$
- $p = 10$
- $b_1 = 1$
- $b_2 = 3$

$$Mo = 60.5 + 10\left(\frac{1}{1+3}\right) = 63$$

Exercise

The following is the statistics scores of economics students at a university.

Class Interval	Frequency (f)
51 – 55	5
56 – 60	6
61 – 65	14
66 – 70	27
71 – 75	21
76 – 80	5
81 – 85	3

What is the mode of the students' statistics scores?