Introduction to Probability

EGI SAFITRI, S.MAT., M.SI

Concept of Probability

Random experiment: an experiment whose possible outcomes can be guessed, but it cannot be known with certainty which possibilities arise.

Experiment is an experiment that can be repeated under the same conditions, while the results are not necessarily the same or Experimentation is a process that produces an outcome (output).

Probability

Event : the outcome of an experiment.

Outcome : the result of observation in an experiment

Sample space : the collection of all possible outcomes of an experiment.

Venn diagram and tree diagram : depiction of sample space

Probability

P(A) = chance (probability) that event A occurs

$0 \leq P(A) \leq 1$

P(A)=0 means that A is unlikely to happen

P(A)=1 means that A must happen

Properties of Probability:

1. $0 \leq P(Ei) \leq 1$

2. $\sum P(Ei) = 1$

Determination of probability value (Conceptual Approach)

1. Classical Probability

When N = the total number of possible outcomes in an experiment, n(A)= the number of outcomes in which event A occurs

$$p(A) = \frac{n(A)}{N}$$

2. Concept of Relative Frequency

If an experiment is conducted n times and event A is observed f times, then :

$$p(A) = \frac{n(A)}{N} = \frac{f}{N}$$

Determination of probability value (Conceptual Approach)

Subjective Probability: Based solely on the feelings, intuition or knowledge of the person determining the probability. Although not a scientific method, this approach can produce fairly accurate probabilities.

Probability Structure (an example)

Experiment :

Recording the US$ exchange rate against the rupiah every Monday, 9am-12pm

Event :

find the exchange rate of US$ against rupiah is less than 10000

Elementry Event:

an event that cannot be broken down into other events.

Probability Structure (an example)

A balanced die is rolled once, looking at the side of the die that comes up.

The sample space ={1,2,3,4,5,6}

Union = “or” = union. Symbol: $\cup$

Intersection = “and” = intersection. Symbol: $\cap$

If X={1,2,3,4,5} and Y={3,5,7,9,11}

then $X \cup Y$ = {1,2,3,4,5,7,9,11} and $X \cap Y$ = {3,5}

Probability Structure

Mutually Exclusive Events are events that have no intersection. That is, one event negates the other; both events cannot occur simultaneously. So:

$$P(X \cap Y) = 0$$

if X and Y *mutually exclusive*.

Probability Structure

Independent Events are events that do not affect each other. This means that the occurrence or non-occurrence of an event does not affect the occurrence or non-occurrence of another event. So:

$$P(X|Y)=P(X)$$

$$P(Y|X)=P(Y)$$

if X and Y are independent events. $P(X|Y)$ means the probability that X occurs if Y is known to have occurred.

Probability Structure

Collectively Exhaustive Events are all the elementary events that may occur in an experiment. So a sample space always consists of Collectively Exhaustive Events.

The complement of an event A is denoted by A' or Ac meaning "not A" is all elementary events in an experiment that are not A. So P(A)+P(Ac)=1

Outcome Calculation Rules

For an operation that can be performed in m ways and a second operation that can be performed in n ways, then both operations can be performed in mn ways. This rule can be extended to three or more operations.

Sampling from a Population

Sampling of size n from a population of size N with replacement will result in :

N^n possibility

Sampling size n from a population of size N without replacement results in ${}_N C_n$ possibility,

$$ {}_N C_n = \binom{N}{n} = \frac{N!}{n!(N-n)!} $$

Multiplication Rule

If an operation can be performed in n1 ways, and if for each of those ways a second operation can be performed in n2 ways, then the two operations together can be performed in n1.n2 ways.

Example

- If two dice are thrown once, then the number of sample points of the sample space is...

The first die has 6 sample points (ways), and for each of the six sample points, the second die also has 6 sample points, so the pair of dice has 6. 6 = 36 ways.2.

- When two coins with sides A and G are thrown once, the number of sample points is 2.2 = 4 ways.

General Multiplication Rule

If an operation can be performed in n1 ways, if for each such way the second operation can be performed in n2 ways, if for each pair of the first two ways the third operation can be performed in n3 ways, and so on then k operations in the sequence can be performed in n1.n2 .n3 ...nk ways.

Example:

Many paths can be taken from the 1st floor to the 4th floor of a building if from the 1st floor to the 2nd floor there are 3 stairs/walks, from the 2nd floor to the 3rd floor there are 2 paths and from the 3rd floor to the 4th floor there are 2 paths, there will be as many as :

3. 2. 2. = 12 ways

Permutations

A permutation is an arrangement formed by all or part of a set of data.There are as many permutations of n different objects as n! (n! = n.(n-1).(n-2)… 2.1

Example:

The number of ways to sit in a row of 4 people is 4! = 4.3.2.1. = 24

Permutations

The number of permutations due to picking r objects from n different objects is :

$$n\,\mathrm{Pr} = P(n,r) = \frac{n!}{(n-r)!}$$

Example:

How many boards consisting of chairman, secretary and treasurer can be formed from 6 candidates?

The number of board members is: $P(6,3) = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6.5.4.3!}{3!} = 120$

Permutations

The number of different permutations of n different objects arranged in a circle is :

$$(n-1)!$$

Example:

How many arrangements are there when 4 bridge card players sit in a circle?

Keeping 1 player in a fixed position, the number of seating arrangements is:

(4-1)! = 3! = 6

Permutations

The number of different permutations of n objects of which n1 is of the first type, n2 is of the second type, …, nk is of the kth type is :

$$\binom{n}{n_1 \quad n_2 \quad \cdots \quad n_k} = \frac{n!}{n_1!n_2!\cdots n_k!}$$

Example:

How many different arrangements are there of a string of decorative lights made up of 2 red, 2 yellow and 3 blue lights?

$$\binom{7}{2 \quad 2 \quad 3} = \frac{7!}{2!2!3!}$$

Combinations

A combination is a picking of r objects from n objects regardless of their order.The number of combinations of r objects from n different objects is:

$$ {}_{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!} $$

The number of boards consisting of 3 people can be composed of 6 people.

$$ {}_{6}C_{3} = \binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6!}{3!3!} = \frac{6.5.4.3!}{3!3.2.1} = 20 $$

Marjinal,union,joint,dan Conditional Probability

Marginal Probability:P(A) = the probability that A occurs

*Union Probability:*P(A∪B) = the probability that A or B occurs

*Joint Probability:*P(A∩B)= the probability that A and B occur

*Conditional Probability:*P(A|B)= the probability that A occurs if it is known that B has occurred

Addition & Multiplication Rules

General rule of addition :

$$P(X \cup Y)=P(X)+P(Y) - P(X \cap Y)$$

Special rule of addition: If X and Y are mutually exclusive, then

$$P(X \cup Y)=P(X)+P(Y)$$

General Rule of Multiplication:

$$P(X \cap Y)=P(X)\ P(Y|X)=P(Y)\ P(X|Y)$$

Special rule of multiplication: If X and Y are independent events, then $P(X \cap Y)=P(X)*P(Y)$

Rules for Conditional Probability

The probability that X occurs if Y is known to have occurred:

$$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X)P(Y \mid X)}{P(Y)}$$

Example

In a city, it is known that: 41% of the population has a motorcycle, 19% have a motorcycle and a car, 22% have a car. Based on the above data, determine:

1. Are motorcycle and car ownership independent?
2. If a resident of the city is randomly drawn, what is the probability that he/she owns a motorcycle and does not own a car?
3. If a resident of the city is randomly picked and it is known that he owns a car, what is the probability that he does not own a motorcycle?
4. If a resident of the city is picked at random, what is the probability that he/she does not own a motorcycle and does not own a car?

Solutions

If S=owns a motorcycle, M=owns a car, then P(S)=0,41; P(SM)=0,19 and P(M)=0,22

1. S and M are independent if and only if P(S)P(M)=P(SM)

   $P(S)P(M)=(0,22)(0,41)=0,0902 \neq P(SM)$

   This means that motorcycle ownership and car ownership in the city are NOT independent.

2. $P(SM^c)=P(S) – P(SM)=0,41 – 0,19=0,22$

3. $P(S^c|M) = \frac{P(S^c \cap M)}{P(M)} = \frac{P(M) - P(M \cap S)}{P(M)}$

$= \frac{0,22 - 0,19}{0,22} = 0,1364$

4. $P(S^c M^c) = 1 - P(S) - P(MS^c)$

$= 1 - 0,41 - 0,03 = 0,56$

Example

Three members of a cooperative are nominated as chairman. The probability that Ali is elected is 0.3; the probability that Badu is elected is 0.5; and the probability that Cokro is elected is 0.2. If Ali is elected, the probability of an increase in cooperative dues is 0.8; if Badu is elected, the probability of an increase in dues is 0.1 and if Cokro is elected, the probability of an increase in dues is 0.4. If a person plans to become a member of a cooperative, but delays it for a few weeks and then a few weeks and finds out that the dues have increased. Find the probability that Cokro will be elected chairman?

Solution

In this problem, suppose :

A1 : Ali who was elected

A2 : Badu who was elected

A3 : Cokro who was elected

B : people who raise dues

Then :

$$P(A_3 \mid B) = \frac{P\left(A_3 \cap B\right)}{P(B)}$$

$$= \frac{P\left(A_3 \cap B\right)}{\sum_{j=1}^{3} P(A_j \cap B)}$$

$$= \frac{P(B \mid A_3).P(A_3)}{P(B \mid A_1).P(A_1) + P(B \mid A_2).P(A_2) + P(B \mid A_3).P(A_3)}$$

$$= \frac{(0,4)(0,2)}{(0,8)(0,3) + (0,1)(0,5) + (0,4)(0,2)}$$

$$= \frac{0,08}{0,24 + 0,05 + 0,08} = \frac{0,08}{0,37} = \frac{8}{37}$$

Expectation (Expected Value)

Definition

Suppose X is a random variable with probability function f(x). The expectation of X is defined as ::

$$E(X) = \begin{cases} \sum x_j P(x_j) & ;\text{X diskrit} \\ \int_{-\infty}^{\infty} x\, f(x)dx & ;\text{X kontinu} \end{cases}$$

The expectation of X is often given the symbol μ or μ_X

Properties of Expectation

If g and h are functions of the random variable X, then :

a. E(c) = c, for c constant

b. E(c g(x)) = c E(g(x))

c. E(c g(x)+d h(x)) = c E(g(x))+d E(h(x))

d. E(c g(x)+d) = c E(g(x))+d

e. if $g(x) \leq h(x)$ then $E(g(x)) \leq E(h(x))$