Pertemuan Ke: 11

TRANSFORMASI Z

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PENGOLAHAN SINYAL DIGITAL

Content

- Overview TZ untuk fungsi eksponensial kausal dan anti kausal, ROC, Zero Pole, TZ fungsi impuls, TZ fungsi sinusoidal
- Overview ITZ: Pecahan Parsial dan Integrasi Kontur, manipulasi ITZ berdasarkan propertynya, ROCnya (kausal dan anti kausal), fungsinya. contoh : ITZ fungsi logaritma $f(z)$ dan TZ fungsi x(n)/n.

Latar Belakang

"Domains of representation "

1. **Domain-n (discrete time):**
   Sequence, impulse response, persamaan beda
2. **Domain-** @ :
   Freq. response, spectral representation
3. **Domain-z**:
   Operator, dan pole-zero

Apabila suatu kasus sulit dipecahkan pada suatu domain tertentu, maka transformasi ke domain yang lain akan menyelesaikannya.

Content

- Transformasi-Z Langsung
- Sifat-sifat Transformasi-Z
- Transformasi-Z Rasional

TRANSFORMASI-Z LANGSUNG

- **Definisi :**

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

- **Contoh 1:**

a. $x_1(n) = \{1, 2, 5, 7, 0, 1\}$

$$X_1(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$$

b. $x_2(n) = \{1, 2, 5, 7, 0, 1\}$

$$X_2(z) = z^2 + 2z^1 + 5 + 7z^{-1} + z^{-3}$$

Contoh 2:

Tentukan transformasi Z dari beberapa sinyal di bawah ini:

a. $x_1(n) = \delta(n)$

b. $x_2(n) = \delta(n-k), k > 0$

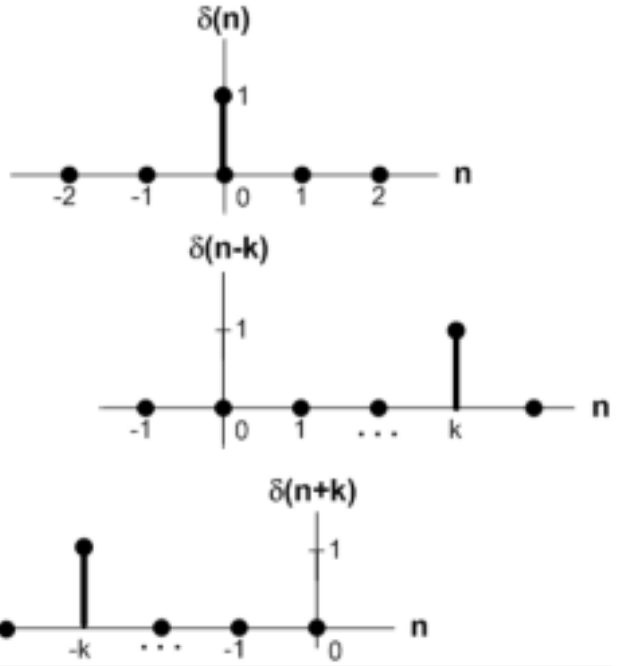
c. $x_3(n) = \delta(n+k), k > 0$

Jawab:

a. $X_1(z) = \sum_{n=-\infty}^{\infty} \delta(n) z^{-n} = 1 \,.\, z^0 = 1$

b. $X_2(z) = \sum_{n=-\infty}^{\infty} \delta(n-k) z^{-n} = z^{-k}$

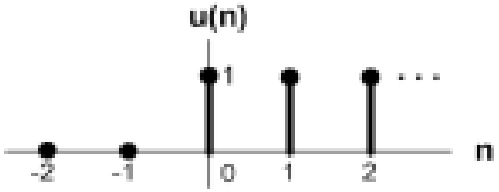
c. $X_3(z) = \sum_{n=-\infty}^{\infty} \delta(n+k) z^{-n} = z^{k}$



- **Contoh 3:**

Tentukan transformasi Z dari sinyal $x(n) = u(n)$

Jawab:



$$X(z) = \sum_{n=0}^{\infty} u(n) z^{-n} = 1 + z^{-1} + z^{-2} + \ldots$$

$$= \frac{1}{1 - z^{-1}} \text{, dimana } \left|z^{-1}\right| < 1 \rightarrow ROC : |z| > 1$$

$$\therefore \quad x(n) = u(n) \quad \rightarrow \quad X(z) = \frac{1}{1 - z^{-1}} \quad , \; ROC : |z| > 1$$

- **Contoh 4:**

Tentukan transformasi Z dari sinyal $x(n) = \alpha^n u(n)$

Jawab:

$$X(z) = \sum_{n=0}^{\infty} \alpha^n u(n) z^{-n} = \sum_{n=0}^{\infty} \left(\alpha z^{-1}\right)^n$$

$$= \sum_{n=0}^{\infty} (A)^n = 1 + A + A^2 + A^3 + \ldots = \frac{1}{1-A}$$

$$= \frac{1}{1-\alpha z^{-1}}, \text{dimana } \left|\alpha z^{-1}\right| < 1 \rightarrow ROC : |z| > \alpha$$

$$\therefore \quad x(n) = \alpha^n u(n) \quad \rightarrow \quad X(z) = \frac{1}{1-\alpha z^{-1}} \quad , ROC : |z| > \alpha$$

TABEL FUNGSI DASAR TZ

Sequence	z-Transform	Region of Convergence
$\delta(n)$	1	all z
$\alpha^n u(n)$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-\alpha^n u(-n-1)$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
$n\alpha^n u(n)$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-n\alpha^n u(-n-1)$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$
$\cos(n\omega_0)u(n)$	$\frac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$\sin(n\omega_0)u(n)$	$\frac{(\sin\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$

SIFAT-SIFAT TZ

Property	Sequence	z-Transform	Region of Convergence
Linearity	$ax(n) + by(n)$	$aX(z) + bY(z)$	Contains $R_x \cap R_y$
Shift	$x(n - n_0)$	$z^{-n_0}X(z)$	R_x
Time reversal	$x(-n)$	$X(z^{-1})$	$1/R_x$
Exponentiation	$\alpha^n x(n)$	$X(\alpha^{-1}z)$	$\lvert\alpha\rvert R_x$
Convolution	$x(n) * y(n)$	$X(z)Y(z)$	Contains $R_x \cap R_y$
Conjugation	$x^*(n)$	$X^*(z^*)$	R_x
Derivative	$nx(n)$	$-z\dfrac{dX(z)}{dz}$	R_x

SIFAT-SIFAT TRANSFORMASI-Z

- **Time Reversal**

$$x(-n) \rightarrow X(z^{-1})$$

- **Contoh 6:**

Tentukan transformasi Z dari sinyal $x(n)= u(-n)$

Jawab:

$$x_1(n)=u(n)\rightarrow X_1(Z)=\frac{1}{1-z^{-1}}, ROC: R_x=|z|>1$$

$$\therefore x(n)=u(-n) \rightarrow X(z)=\frac{1}{1-\left(z^{-1}\right)^{-1}}=\frac{1}{1-z}, ROC: R_x=|z|<1$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Time Reversal**

$$x(-n) \quad \rightarrow \quad X(z^{-1})$$

- **Contoh 6:**

Tentukan transformasi Z dari sinyal $x(n)=u(-n)$

Jawab:

$$x_1(n)=u(n)\rightarrow X_1(Z)=\frac{1}{1-z^{-1}}, ROC: R_x=|z|>1$$

$$\therefore\ x(n)=u(-n)\rightarrow X(z)=\frac{1}{1-\left(z^{-1}\right)^{-1}}=\frac{1}{1-z},\ ROC:\ {}^{1}\!/_{R_x}=|z|<1$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Diferensiasi dalam domain z**

$$nx(n) \quad \rightarrow \quad -z\frac{dX(z)}{dz}$$

- **Contoh 7:**

Tentukan transformasi Z dari sinyal $x(n) = n\,a^n u(n)$

Jawab:

$$x_1(n) = a^n u(n) \quad \rightarrow \quad X_1(z) = \frac{1}{1-az^{-1}}, ROC: R_x = |z| > a$$

$$\therefore x(n) = n\,a^n u(n) \quad \rightarrow \quad X(z) = -z\frac{dX_1(z)}{dz} = -z\frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right)$$

$$\therefore n\,a^n u(n) \quad \rightarrow \quad \frac{az^{-1}}{\left(1-az^{-1}\right)^2}$$

$$= (-z)\frac{-az^{-2}}{\left(1-az^{-1}\right)^2} = \frac{az^{-1}}{\left(1-az^{-1}\right)^2}$$

SEKIAN.....
See You Next Meeting
Thank's

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