

Materi

Capaian

Topik: Analisis Fungsi Real

Subtopik:

- Daerah Asal
- Fungsi dan Grafik
- Fungsi Genap dan Ganjil
- **Fungsi Trigonometri**
- **Menggambar Fungsi dengan Teknik Pergeseran**
- Fungsi Komposisi
- Daerah Asal dan Daerah Nilai Fungsi Komposisi

Latihan Soal



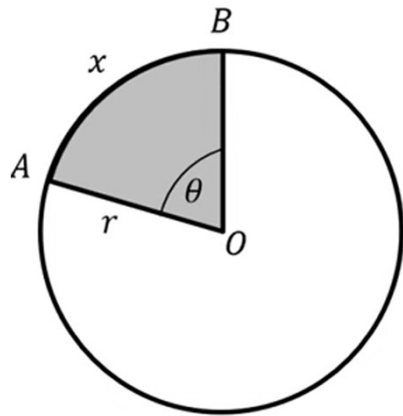
Capaian

Mahir menentukan **domain** dan **range**, menggambar grafik fungsi **linier** serta **kuadrat** menggunakan teknik **pergeseran**, mengidentifikasi fungsi **genap/ganjil**, dan menguasai operasi **komposisi** dua fungsi atau lebih untuk menyelesaikan persoalan kalkulus.



Fungsi Trigonometri (1)

Ukuran Sudut dan Radian

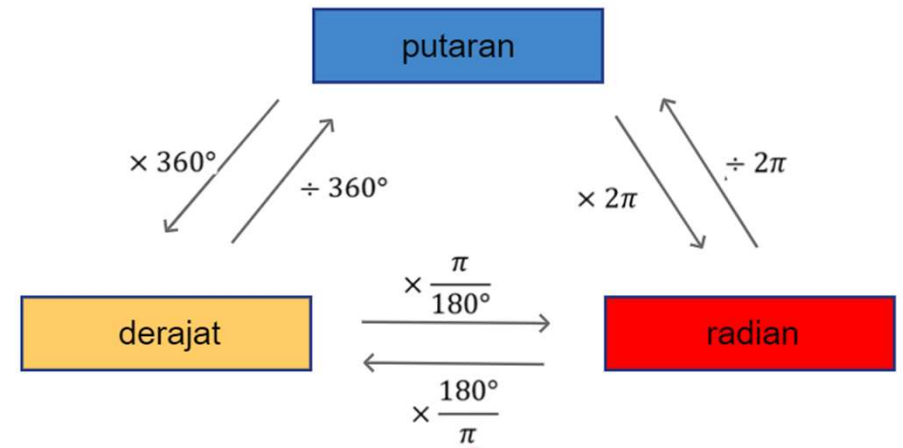


$$\theta(\text{rad}) = \frac{x}{r}$$

x = Panjang busur yang diapit
 r = jari-jari

$$1 \text{ putaran} = 360^\circ = 2\pi \text{ rad}$$

$$\frac{1}{2} \text{ putaran} = 180^\circ = \pi \text{ rad}$$

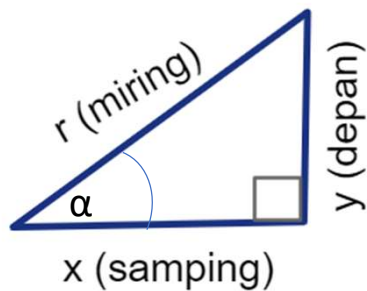


Putaran	Derajat	Radian
$\frac{1}{2}$ putaran		
	60°	
		$\frac{5}{6} \pi \text{ rad}$
$\frac{7}{8}$ putaran		



Fungsi Trigonometri (2)

Perbandingan Trigonometri



1. Sinus (sin)
2. Kosinus (cos)
3. Tangen (tag)
4. Kotangen (cotan)
5. Sekan (sec)
6. Kosekan (cosec)

$$\sin \alpha = \frac{\text{depan}}{\text{miring}} = \frac{y}{r}$$

$$\text{cosec } \alpha = \frac{\text{miring}}{\text{depan}} = \frac{r}{y}$$

$$\cos \alpha = \frac{\text{samping}}{\text{miring}} = \frac{x}{r}$$

$$\sec \alpha = \frac{\text{miring}}{\text{samping}} = \frac{r}{x}$$

$$\tan \alpha = \frac{\text{depan}}{\text{samping}} = \frac{y}{x}$$

$$\cotan \alpha = \frac{\text{samping}}{\text{depan}} = \frac{x}{y}$$

Perbandingan Trigonometri

$$\text{cosec } \alpha = \frac{1}{\sin \alpha}$$

$$\sec \alpha = \frac{1}{\cos \alpha}$$

$$\cotan \alpha = \frac{1}{\tan \alpha}$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\cotan \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\tan^2 \alpha + 1 = \sec^2 \alpha$$

$$\cotan^2 \alpha + 1 = \text{cosec}^2 \alpha$$

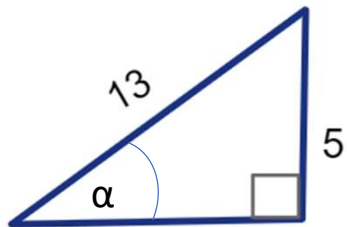


Fungsi Trigonometri (3)

□ Soal (1) – Perbandingan trigonometri

Diketahui $\sin \alpha = \frac{5}{13}$, dengan α sudut lancip. Tentukan nilai perbandingan trigonometri lainnya:

- 1) Dengan menggambar segitiga siku siku bantuan
- 2) Dengan rumus perbandingan trigonometri



$$r^2 = x^2 + y^2$$

$$x = \sqrt{r^2 - y^2} = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$$

$$1) \quad \sin \alpha = \frac{5}{13}$$

$$\cos \alpha = \frac{12}{13}$$

$$\tan \alpha = \frac{5}{12}$$

$$2) \quad \operatorname{cosec} \alpha = \frac{13}{5}$$

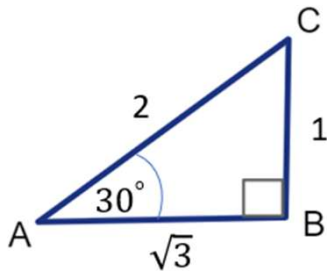
$$\sec \alpha = \frac{13}{12}$$

$$\operatorname{cotan} \alpha = \frac{12}{5}$$

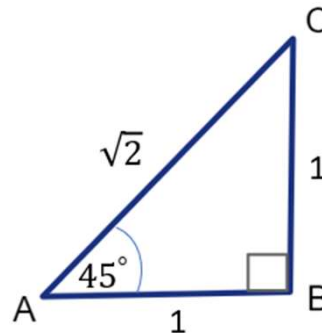


Fungsi Trigonometri (4)

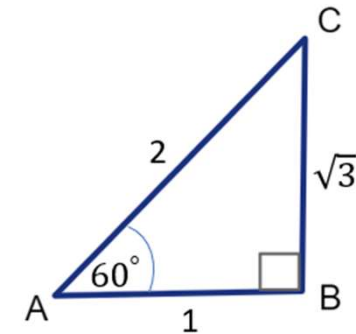
Perbandingan Trigonometri Sudut Istimewa



$$\begin{aligned} \sin 30 &= \frac{1}{2} & \operatorname{cosec} 30 &= \frac{2}{1} = 2 \\ \cos 30 &= \frac{\sqrt{3}}{2} & \sec 30 &= \frac{2}{\sqrt{3}} \\ \tan 30 &= \frac{1}{\sqrt{3}} & \operatorname{cotan} 30 &= \frac{\sqrt{3}}{1} = \sqrt{3} \end{aligned}$$



$$\begin{aligned} \sin 45 &= \frac{1}{\sqrt{2}} & \operatorname{cosec} 45 &= \frac{\sqrt{2}}{1} = \sqrt{2} \\ \cos 45 &= \frac{1}{\sqrt{2}} & \sec 45 &= \frac{\sqrt{2}}{1} = \sqrt{2} \\ \tan 45 &= \frac{1}{1} = 1 & \operatorname{cotan} 45 &= \frac{1}{1} = 1 \end{aligned}$$



$$\begin{aligned} \sin 60 &= \frac{\sqrt{3}}{2} & \operatorname{cosec} 60 &= \frac{2}{\sqrt{3}} \\ \cos 60 &= \frac{1}{2} & \sec 60 &= \frac{1}{2} \\ \tan 60 &= \frac{\sqrt{3}}{1} & \operatorname{cotan} 60 &= \frac{1}{\sqrt{3}} \end{aligned}$$



Fungsi Trigonometri (5)

□ Perbandingan Trigonometri Sudut Istimewa

α	0°	30°	45°	60°	90°
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\cos \alpha$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
$\tan \alpha$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	<i>TD</i>
$\cotan \alpha$	<i>TD</i>	$\sqrt{3}$	1	$\frac{1}{3}\sqrt{3}$	0
$\sec \alpha$	1	$\frac{2}{3}\sqrt{3}$	$\sqrt{2}$	2	<i>TD</i>
$\operatorname{cosec} \alpha$	<i>TD</i>	2	$\sqrt{2}$	$\frac{2}{3}\sqrt{3}$	1

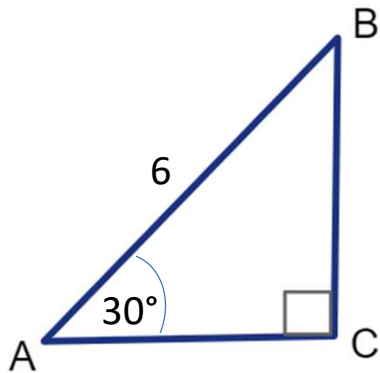


Fungsi Trigonometri (6)

Soal (2) – Perbandingan trigonometri sudut istimewa

Diketahui segitiga ABC, siku-siku di C. Jika diketahui panjang AB = 6 cm dan sudut BAC = 30°. Tentukan:

- 1) Panjang AC
- 2) Panjang BC



1) Panjang AC

$$\cos 30 = \frac{AC}{6}$$

$$\frac{\sqrt{3}}{2} = \frac{AC}{6}$$

$$AC = \frac{6\sqrt{3}}{2} = 3\sqrt{3} \text{ cm}$$

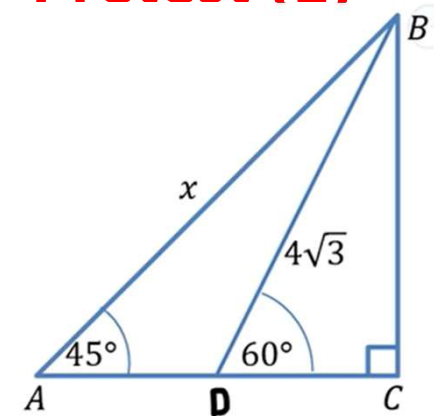
2) Panjang BC

$$\sin 30 = \frac{BC}{6}$$

$$\frac{1}{2} = \frac{BC}{6}$$

$$BC = \frac{6}{2} = 3 \text{ cm}$$

Pretest (1)

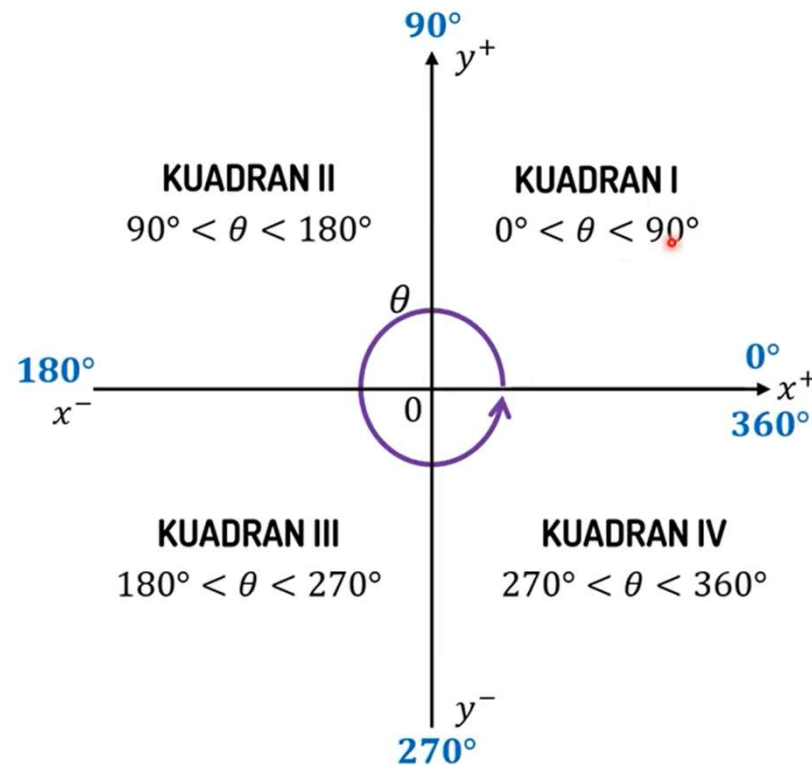


Berapa nilai x?



Fungsi Trigonometri (7)

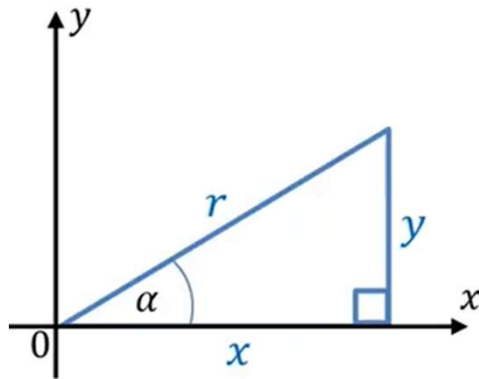
□ Perbandingan Trigonometri Sudut Berelasi





Fungsi Trigonometri (8)

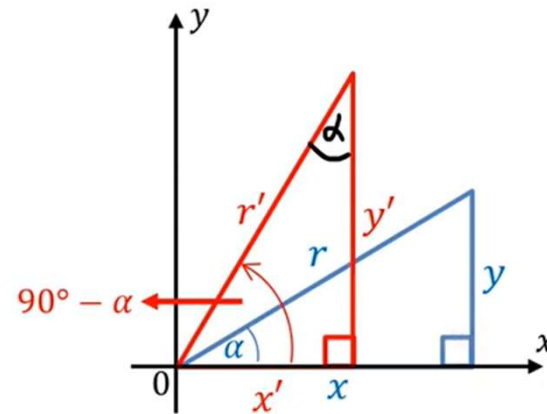
Perbandingan Trigonometri Sudut Berelasi Kuadran I



$$\sin \alpha = \frac{y}{r} \quad \text{cosec } \alpha = \frac{r}{y}$$

$$\cos \alpha = \frac{x}{r} \quad \text{sec } \alpha = \frac{r}{x}$$

$$\tan \alpha = \frac{y}{x} \quad \text{cotan } \alpha = \frac{x}{y}$$



$$\begin{aligned} x' &= y \\ y' &= x \\ r' &= r \end{aligned}$$

$$\sin (90 - \alpha) = \frac{y'}{r'} = \frac{x}{r} = \cos \alpha$$

$$\text{cosec } (90 - \alpha) = \frac{r'}{y'} = \frac{r}{x} = \sec \alpha$$

$$\cos (90 - \alpha) = \frac{x'}{r'} = \frac{y}{r} = \sin \alpha$$

$$\sec (90 - \alpha) = \frac{r'}{x'} = \frac{r}{y} = \text{cosec } \alpha$$

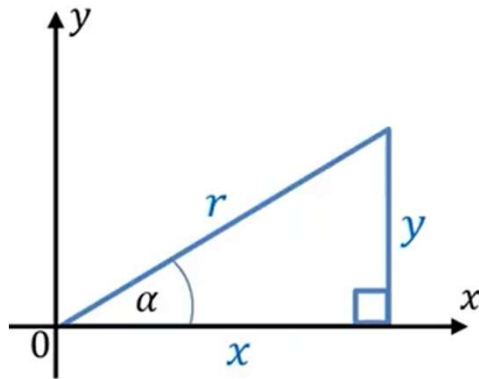
$$\tan (90 - \alpha) = \frac{y'}{x'} = \frac{x}{y} = \text{cotan } \alpha$$

$$\text{cotan } (90 - \alpha) = \frac{x'}{y'} = \frac{y}{x} = \tan \alpha$$



Fungsi Trigonometri (9)

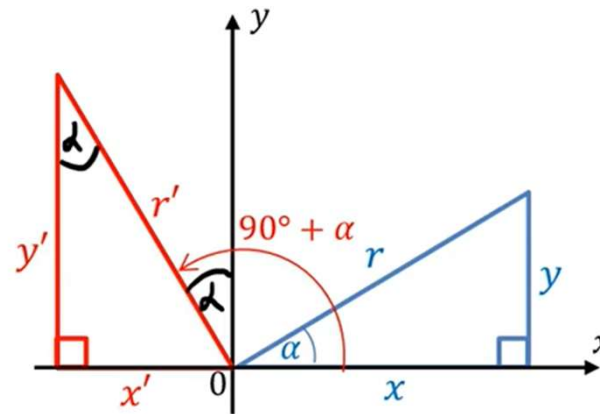
Perbandingan Trigonometri Sudut Berelasi Kuadran II



$$\sin \alpha = \frac{y}{r} \quad \text{cosec } \alpha = \frac{r}{y}$$

$$\cos \alpha = \frac{x}{r} \quad \text{sec } \alpha = \frac{r}{x}$$

$$\tan \alpha = \frac{y}{x} \quad \text{cotan } \alpha = \frac{x}{y}$$



$$\begin{aligned} x' &= -y \\ y' &= x \\ r' &= r \end{aligned}$$

$$\sin (90 + \alpha) = \frac{y'}{r'} = \frac{x}{r} = \cos \alpha$$

$$\cos (90 + \alpha) = \frac{x'}{r'} = \frac{-y}{r} = -\sin \alpha$$

$$\tan (90 + \alpha) = \frac{y'}{x'} = \frac{x}{-y} = -\cotan \alpha$$

$$\text{cosec } (90 + \alpha) = \frac{r'}{y'} = \frac{r}{x} = \sec \alpha$$

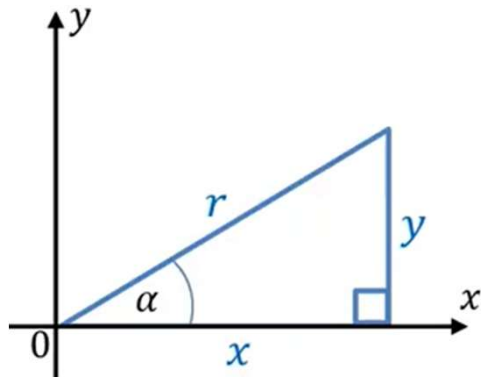
$$\text{sec } (90 + \alpha) = \frac{r'}{x'} = \frac{r}{-y} = -\text{cosec } \alpha$$

$$\text{cotan } (90 + \alpha) = \frac{x'}{y'} = \frac{-y}{x} = -\tan \alpha$$

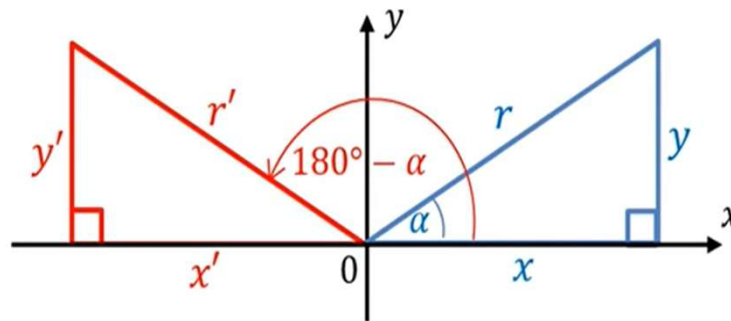


Fungsi Trigonometri (10)

Perbandingan Trigonometri Sudut Berelasi Kuadran II



$$\begin{aligned} x' &= -x \\ y' &= y \\ r' &= r \end{aligned}$$



$$\sin \alpha = \frac{y}{r} \quad \text{cosec } \alpha = \frac{r}{y}$$

$$\sin (180 - \alpha) = \frac{y'}{r'} = \frac{y}{r} = \sin \alpha$$

$$\text{cosec } (180 - \alpha) = \frac{r'}{y'} = \frac{r}{y} = \text{cosec } \alpha$$

$$\cos \alpha = \frac{x}{r} \quad \sec \alpha = \frac{r}{x}$$

$$\cos (180 - \alpha) = \frac{x'}{r'} = \frac{-x}{r} = -\cos \alpha$$

$$\sec (180 - \alpha) = \frac{r'}{x'} = \frac{r}{-x} = -\sec \alpha$$

$$\tan \alpha = \frac{y}{x} \quad \cotan \alpha = \frac{x}{y}$$

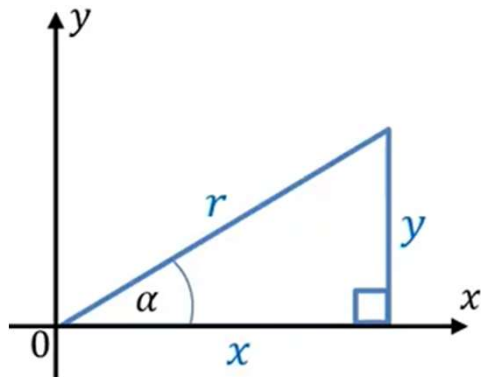
$$\tan (180 - \alpha) = \frac{y'}{x'} = \frac{y}{-x} = -\tan \alpha$$

$$\cotan (180 - \alpha) = \frac{x'}{y'} = \frac{-x}{y} = -\cotan \alpha$$



Fungsi Trigonometri (11)

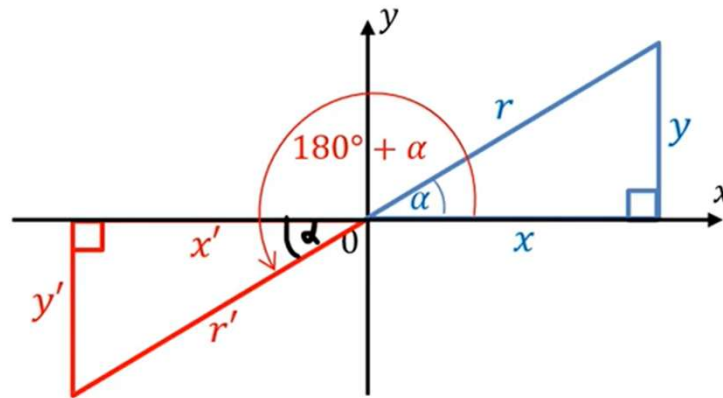
Perbandingan Trigonometri Sudut Berelasi Kuadran III



$$\sin \alpha = \frac{y}{r} \quad \text{cosec } \alpha = \frac{r}{y}$$

$$\cos \alpha = \frac{x}{r} \quad \text{sec } \alpha = \frac{r}{x}$$

$$\tan \alpha = \frac{y}{x} \quad \text{cotan } \alpha = \frac{x}{y}$$



$$\sin (180 + \alpha) = \frac{y'}{r'} = \frac{-y}{r} = -\sin \alpha$$

$$\cos (180 + \alpha) = \frac{x'}{r'} = \frac{-x}{r} = -\cos \alpha$$

$$\tan (180 + \alpha) = \frac{y'}{x'} = \frac{-y}{-x} = \tan \alpha$$

$$\begin{aligned} x' &= -x \\ y' &= -y \\ r' &= r \end{aligned}$$

$$\text{cosec } (180 + \alpha) = \frac{r'}{y'} = \frac{r}{-y} = -\text{cosec } \alpha$$

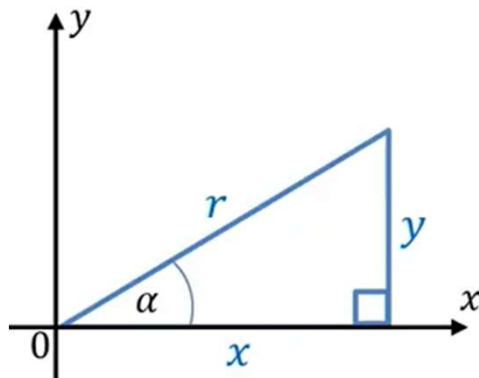
$$\text{sec } (180 + \alpha) = \frac{r'}{x'} = \frac{r}{-x} = -\text{sec } \alpha$$

$$\text{cotan } (180 + \alpha) = \frac{x'}{y'} = \frac{-x}{-y} = \text{cotan } \alpha$$

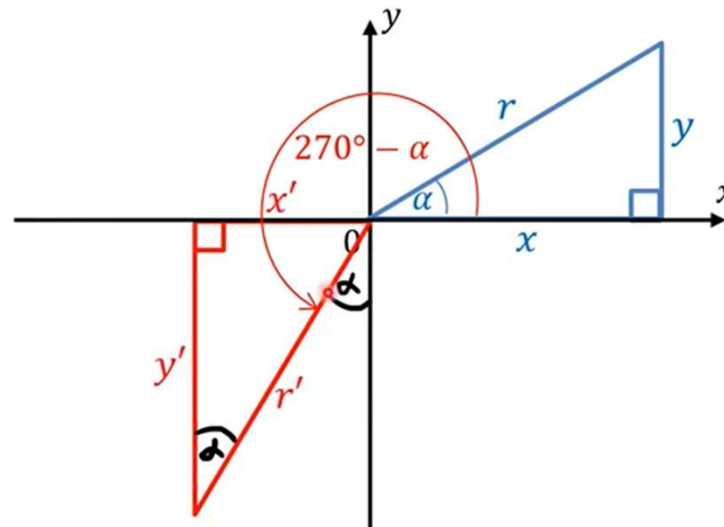


Fungsi Trigonometri (12)

Perbandingan Trigonometri Sudut Berelasi Kuadran III



$$\begin{aligned} \sin \alpha &= \frac{y}{r} & \operatorname{cosec} \alpha &= \frac{r}{y} \\ \cos \alpha &= \frac{x}{r} & \sec \alpha &= \frac{r}{x} \\ \tan \alpha &= \frac{y}{x} & \operatorname{cotan} \alpha &= \frac{x}{y} \end{aligned}$$



$$\begin{aligned} \sin (270 - \alpha) &= \frac{y'}{r'} = \frac{-x}{r} = -\cos \alpha \\ \cos (270 - \alpha) &= \frac{x'}{r'} = \frac{-y}{r} = -\sin \alpha \\ \tan (270 - \alpha) &= \frac{y'}{x'} = \frac{-x}{-y} = \operatorname{cotan} \alpha \end{aligned}$$

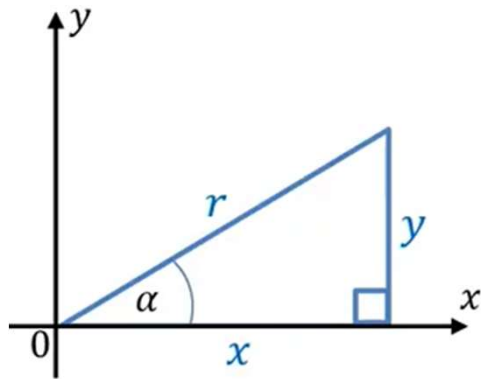
$$\begin{aligned} x' &= -y \\ y' &= -x \\ r' &= r \end{aligned}$$

$$\begin{aligned} \operatorname{cosec} (270 - \alpha) &= \frac{r'}{y'} = \frac{r}{-x} = -\sec \alpha \\ \sec (270 - \alpha) &= \frac{r'}{x'} = \frac{r}{-y} = -\operatorname{cosec} \alpha \\ \operatorname{cotan} (270 - \alpha) &= \frac{x'}{y'} = \frac{-y}{-x} = \tan \alpha \end{aligned}$$

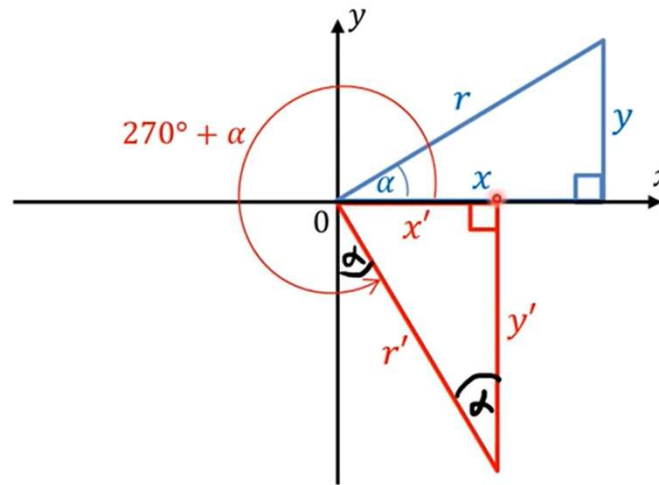


Fungsi Trigonometri (13)

Perbandingan Trigonometri Sudut Berelasi Kuadran IV



$$\begin{aligned} \sin \alpha &= \frac{y}{r} & \operatorname{cosec} \alpha &= \frac{r}{y} \\ \cos \alpha &= \frac{x}{r} & \sec \alpha &= \frac{r}{x} \\ \tan \alpha &= \frac{y}{x} & \operatorname{cotan} \alpha &= \frac{x}{y} \end{aligned}$$



$$\sin (270 + \alpha) = \frac{y'}{r'} = \frac{-x}{r} = -\cos \alpha$$

$$\cos (270 + \alpha) = \frac{x'}{r'} = \frac{y}{r} = \sin \alpha$$

$$\tan (270 + \alpha) = \frac{y'}{x'} = \frac{-x}{y} = -\operatorname{cotan} \alpha$$

$$\begin{aligned} x' &= y \\ y' &= -x \\ r' &= r \end{aligned}$$

$$\operatorname{cosec} (270 + \alpha) = \frac{r'}{y'} = \frac{r}{-x} = -\sec \alpha$$

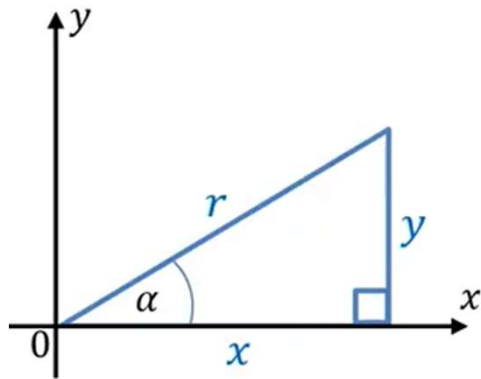
$$\sec (270 + \alpha) = \frac{r'}{x'} = \frac{r}{y} = \operatorname{cosec} \alpha$$

$$\operatorname{cotan} (270 + \alpha) = \frac{x'}{y'} = \frac{y}{-x} = -\tan \alpha$$

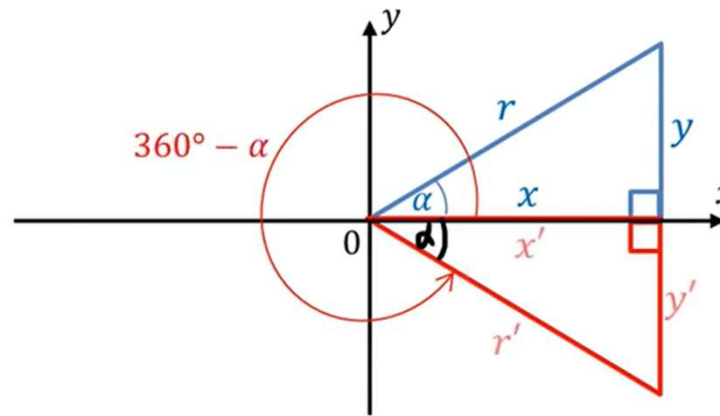


Fungsi Trigonometri (14)

Perbandingan Trigonometri Sudut Berelasi Kuadran IV



$$\begin{aligned} \sin \alpha &= \frac{y}{r} & \operatorname{cosec} \alpha &= \frac{r}{y} \\ \cos \alpha &= \frac{x}{r} & \sec \alpha &= \frac{r}{x} \\ \tan \alpha &= \frac{y}{x} & \operatorname{cotan} \alpha &= \frac{x}{y} \end{aligned}$$



$$\begin{aligned} \sin (360 - \alpha) &= \frac{y'}{r'} = \frac{-y}{r} = -\sin \alpha \\ \cos (360 - \alpha) &= \frac{x'}{r'} = \frac{x}{r} = \cos \alpha \\ \tan (360 - \alpha) &= \frac{y'}{x'} = \frac{-y}{x} = -\tan \alpha \end{aligned}$$

$$\begin{aligned} x' &= x \\ y' &= -y \\ r' &= r \end{aligned}$$

$$\begin{aligned} \operatorname{cosec} (360 - \alpha) &= \frac{r'}{y'} = \frac{r}{-y} = -\operatorname{cosec} \alpha \\ \sec (360 - \alpha) &= \frac{r'}{x'} = \frac{r}{x} = \sec \alpha \\ \operatorname{cotan} (360 - \alpha) &= \frac{x'}{y'} = \frac{x}{-y} = -\operatorname{cotan} \alpha \end{aligned}$$



Fungsi Trigonometri (15)

□ Soal (3) – Perbandingan trigonometri sudut berelasi

Tentukan nilai dari perbandingan trigonometri berikut: $\tan 240^\circ$, $\sin 315^\circ$, $\cos 180^\circ$, $\operatorname{cosec} \frac{5}{4} \pi$, $\cotan \frac{4}{3} \pi$, $\sec \frac{3}{2} \pi$

1) Sudut 240° berada pada kuadran III

$$\tan (270 - \alpha) = \cotan \alpha$$

$$\tan (270 - 30) = \cotan 30 = \sqrt{3}$$

2) Sudut 315° berada pada kuadran IV

$$\sin (360 - \alpha) = -\sin \alpha$$

$$\sin (360 - 45) = -\sin 45 = -\frac{1}{2}\sqrt{2}$$

3) Sudut 180° berada pada kuadran II

$$\cos (180 - \alpha) = -\cos \alpha$$

$$\cos (180 - 0) = -\cos 0 = -1$$

4) $\operatorname{cosec} \frac{5}{4} \pi$ diubah ke derajat

$$\operatorname{cosec} \left(\frac{5}{4} \pi \cdot \frac{180}{\pi} \right) = \operatorname{cosec} 225$$

$$\operatorname{cosec} (180 + 45) = \operatorname{cosec} 45 = -\sqrt{2}$$

5) $\cotan \frac{4}{3} \pi$ diubah ke derajat

$$\cotan \left(\frac{4}{3} \pi \cdot \frac{180}{\pi} \right) = -\sin 240$$

$$\cotan (180 + 60) = \cotan 60 = \frac{1}{3}\sqrt{3}$$

6) $\sec \frac{3}{2} \pi$ diubah ke derajat

$$\sec \left(\frac{3}{2} \pi \cdot \frac{180}{\pi} \right) = \sec 270$$

$$\sec (180 + 90) = \sec 90 = \text{TD}$$



Fungsi Trigonometri (16)

Soal (4) – Perbandingan trigonometri sudut berelasi kurang/ lebih dari 360

Tentukan nilai dari perbandingan trigonometri berikut: $\cotan(-45^\circ)$, $\cos(-120^\circ)$, $\operatorname{cosec}(810^\circ)$, $\operatorname{cosec}\frac{5}{4}\pi$, $\cotan\frac{4}{3}\pi$, $\sec\frac{3}{2}\pi$

1) $\cotan(-45^\circ)$

$$= \cotan(-45 + 360)$$

$$= \cotan 315$$

$$= \cotan (360 - 45)$$

$$= \cotan 45 = -1$$

2) $\cos(-120^\circ)$

$$= \cos(-120 + 360)$$

$$= \cos(240)$$

$$= \cos(180 + 60)$$

$$= \cos 60 = -\frac{1}{2}$$

3) Sudut 180° berada pada kuadran II

$$\cos (180 - \alpha) = -\cos \alpha$$

$$\cos (180 - 0) = -\cos 0 = -1$$

4) $\operatorname{cosec}\frac{5}{4}\pi$ diubah ke derajat

$$\operatorname{cosec}\left(\frac{5}{4}\pi \cdot \frac{180}{\pi}\right) = \operatorname{cosec} 225$$

$$\operatorname{cosec} (180 + 45) = \operatorname{cosec} 45 = -\sqrt{2}$$

5) $\cotan\frac{4}{3}\pi$ diubah kederajat

$$\cotan\left(\frac{4}{3}\pi \cdot \frac{180}{\pi}\right) = -\sin 240$$

$$\cotan (180 + 60) = \cotan 60 = \frac{1}{3}\sqrt{3}$$

6) $\sec\frac{3}{2}\pi$ diubah kederajat

$$\sec\left(\frac{3}{2}\pi \cdot \frac{180}{\pi}\right) = \sec 270$$

$$\sec (180 + 90) = \sec 90 = \text{TD}$$



Fungsi Trigonometri (17)

Identitas Trigonometri Dasar

NO.	IDENTITAS TRIGONOMETRI	JENIS
1.	$\operatorname{cosec} \alpha = \frac{1}{\sin \alpha} \leftrightarrow \sin \alpha = \frac{1}{\operatorname{cosec} \alpha}$	Identitas Kebalikan
2.	$\sec \alpha = \frac{1}{\cos \alpha} \leftrightarrow \cos \alpha = \frac{1}{\sec \alpha}$	
3.	$\operatorname{cotan} \alpha = \frac{1}{\tan \alpha} \leftrightarrow \tan \alpha = \frac{1}{\operatorname{cotan} \alpha}$	
4.	$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$	Identitas Perbandingan
5.	$\operatorname{cotan} \alpha = \frac{\cos \alpha}{\sin \alpha}$	
6.	$\sin^2 \alpha + \cos^2 \alpha = 1$	Identitas Pythagoras
7.	$\tan^2 \alpha + 1 = \sec^2 \alpha$	
8.	$\operatorname{cotan}^2 \alpha + 1 = \operatorname{cosec}^2 \alpha$	

- Cara membuktikan identitas trigonometri: mengubah salah satu ruas yang lebih kompleks (biasanya ruas kiri) menjadi ruas lainnya yang lebih sederhana (biasanya ruas kanan)
- Cara mengubah salah satu ruas tersebut adalah dengan menggunakan identitas trigonometri dasar dan/atau rumus-rumus aljabar

NO.	RUMUS ALJABAR
1.	$(a + b)^2 = a^2 + 2ab + b^2$
2.	$(a - b)^2 = a^2 - 2ab + b^2$
3.	$a^2 - b^2 = (a + b)(a - b)$
4.	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
5.	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

TIPS MEMBUKTIKAN IDENTITAS TRIGONOMETRI

- ✓ Mengubah bentuk tan, cotan, sec, dan cosec menjadi bentuk yang mengandung sin dan cos
- ✓ Menyamakan penyebut
- ✓ Memfaktorkan
- ✓ Mengalikan dengan sekawan

- Soal (5) : Buktikan $(\sin \alpha + \cos \alpha)^2 - 2 \sin \alpha \cos \alpha = 1$

Pembuktian: $(\sin \alpha + \cos \alpha)^2 - 2 \sin \alpha \cos \alpha = 1$

$$\sin^2 \alpha + \cancel{2 \sin \alpha \cos \alpha} + \cos^2 \alpha - \cancel{2 \sin \alpha \cos \alpha} = 1$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \text{terbukti sesuai dengan identitas trigonometri no 6}$$



Fungsi Trigonometri (18)

□ Menggambar Fungsi dengan Lingkaran Satuan, $y = \sin x$

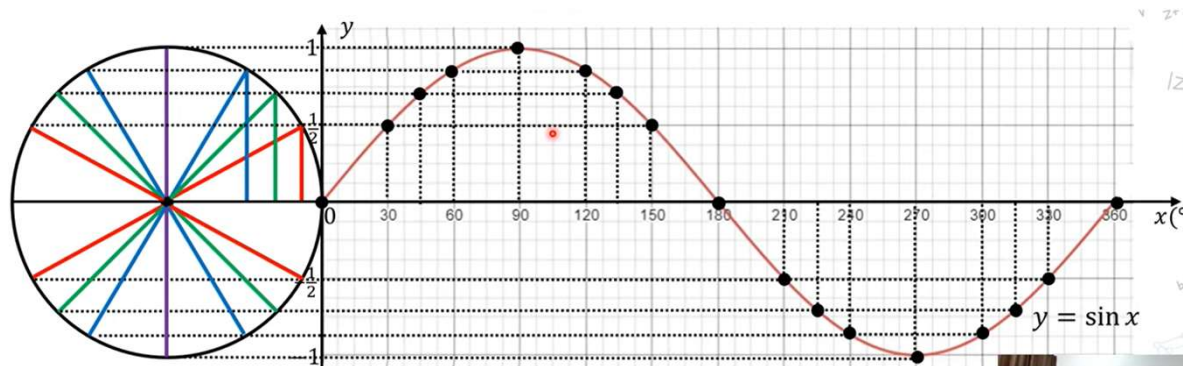
- Buatlah grafik fungsi $f(x) = \sin x$, untuk $0^\circ \leq x \leq 360^\circ$

Cara:

1) Membuat tabel

x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
y	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{3}$	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0

2) Membuat bidang koordinat



- Nilai max : 1
- Nilai min : -1
- Amplitudo : 1
- Periode : 360°

$$D_f = \{x \mid 0^\circ \leq x \leq 360^\circ, x \in \mathbb{R}\}$$

$$R_f = \{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$$



Fungsi Trigonometri (19)

□ Menggambar Fungsi dengan Lingkaran Satuan, $y = \cos x$

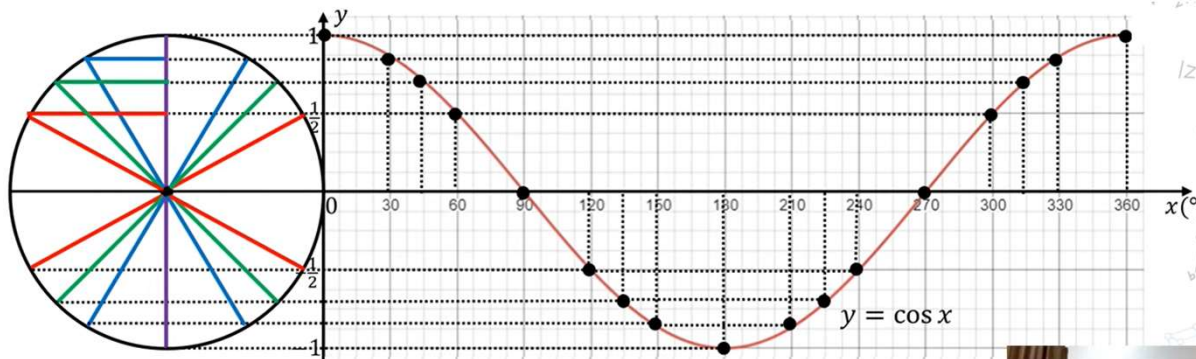
- Buatlah grafik fungsi $f(x) = \cos x$, untuk $0^\circ \leq x \leq 360^\circ$

Cara:

1) Membuat tabel

x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
y	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{3}$	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1

2) Membuat bidang koordinat



- Nilai max : 1
- Nilai min : -1
- Amplitudo : 1
- Periode : 360°

$$D_f = \{x \mid 0^\circ \leq x \leq 360^\circ, x \in \mathbb{R}\}$$

$$R_f = \{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$$



Fungsi Trigonometri (20)

□ Menggambar Fungsi dengan Lingkaran Satuan, $y = \tan x$

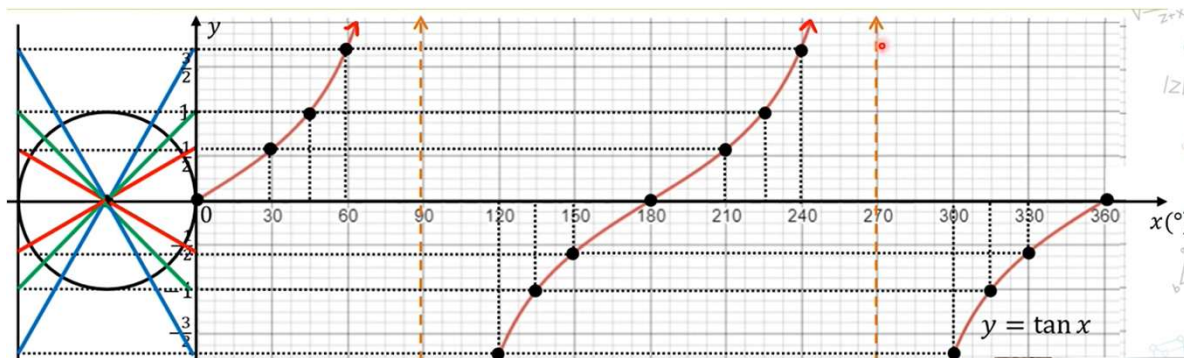
- Buatlah grafik fungsi $f(x) = \tan x$, untuk $0^\circ \leq x \leq 360^\circ$

Cara:

1) Membuat tabel

x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
y	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	TD	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	TD	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0

2) Membuat bidang koordinat



- Nilai max : ∞
- Nilai min : $-\infty$
- Amplitudo : ∞
- Periode : 180°

$$D_f = \{x \mid 0^\circ \leq x \leq 360^\circ, x \neq 90, x \neq 270, x \in \mathbb{R}\}$$

$$R_f = \{y \mid y \in \mathbb{R}\}$$



Fungsi Trigonometri (21)

□ Transformasi Grafik Fungsi Trigonometri Dasar

BENTUK UMUM

$$y = A \sin B(x + C) + D$$

$$y = A \cos B(x + C) + D$$

$$y = A \tan B(x + C) + D$$

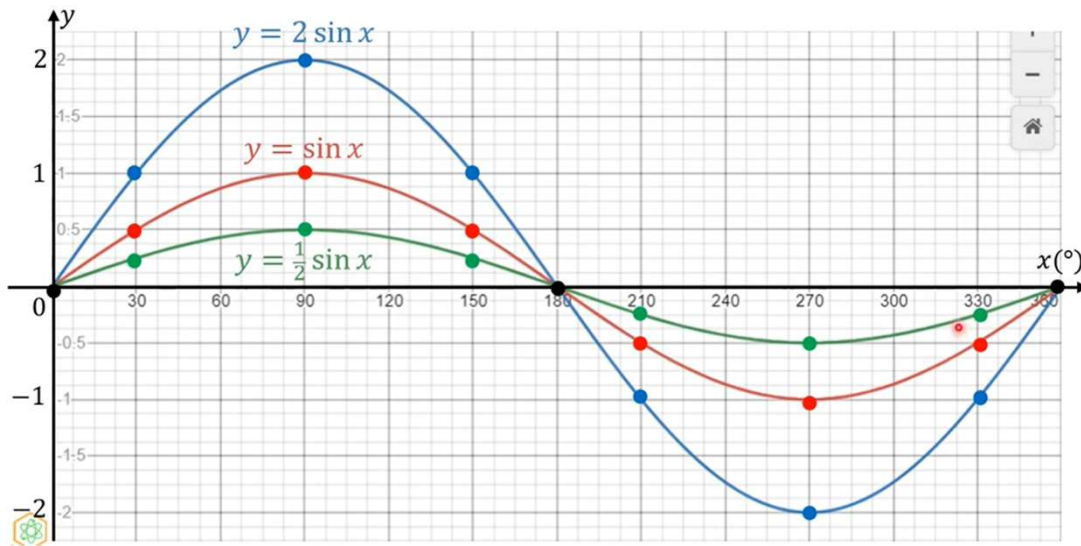
Jenis Transformasi:

- 1) Peregangan vertical dan pencerminan terhadap sumbu x (A)
- 2) Peregangan horizontal (B)
- 3) Pergeseran horizontal (C)
- 4) Pergeseran vertikal (D)



Fungsi Trigonometri (22)

□ Peregangan Vertikal dan Pencerminan terhadap sumbu x (A)



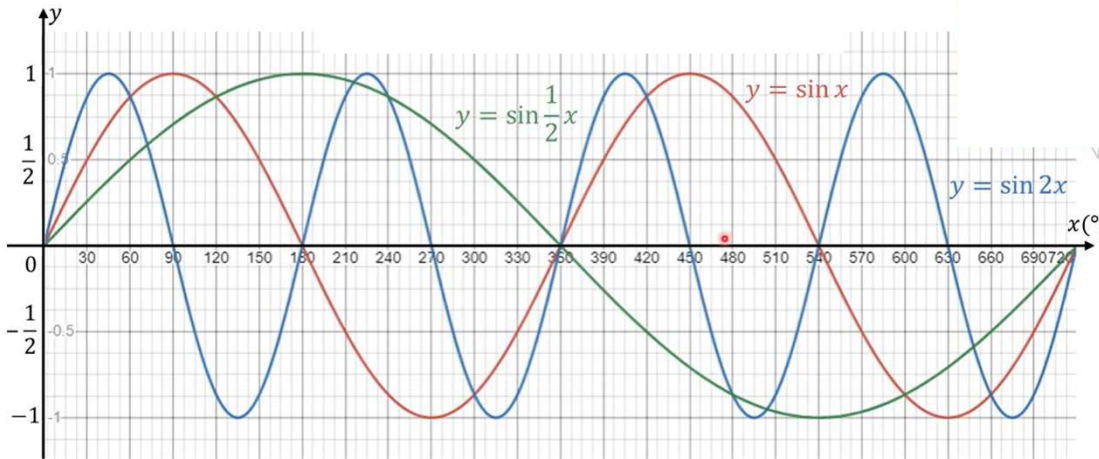
KARAKTERISTIK	$y = \sin x$	$y = 2 \sin x$	$y = \frac{1}{2} \sin x$
Nilai maksimum	1	2	$\frac{1}{2}$
Nilai minimum	-1	-2	$-\frac{1}{2}$
Amplitudo	1	2	$\frac{1}{2}$
Periode	360°	360°	360°

A



Fungsi Trigonometri (23)

□ Peregangan Horizontal (B)



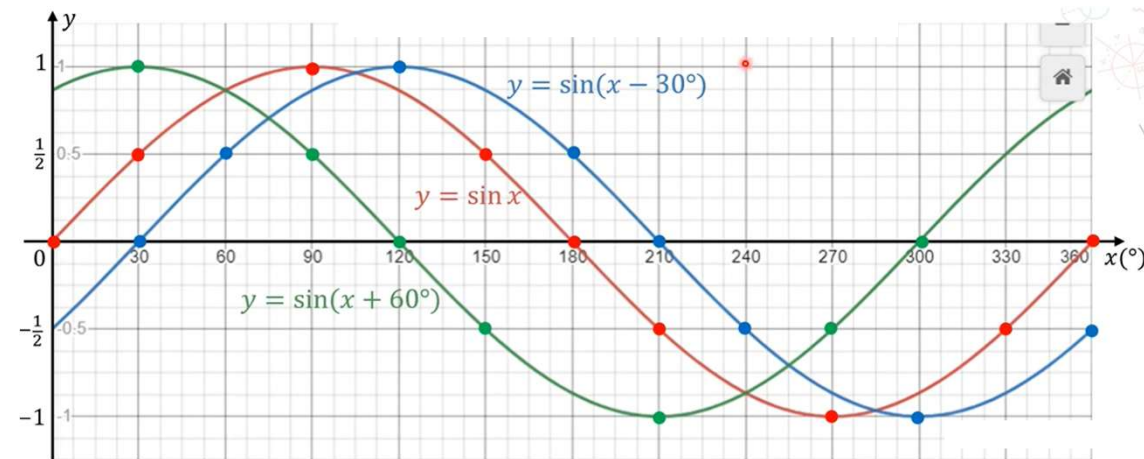
KARAKTERISTIK	$y = \sin x$	$y = \sin 2x$	$y = \sin \frac{1}{2}x$
Nilai maksimum	1	1	1
Nilai minimum	-1	-1	-1
Amplitudo	1	1	1
Periode	360°	180°	720°

B



Fungsi Trigonometri (24)

□ Pergeseran Horizontal (C)

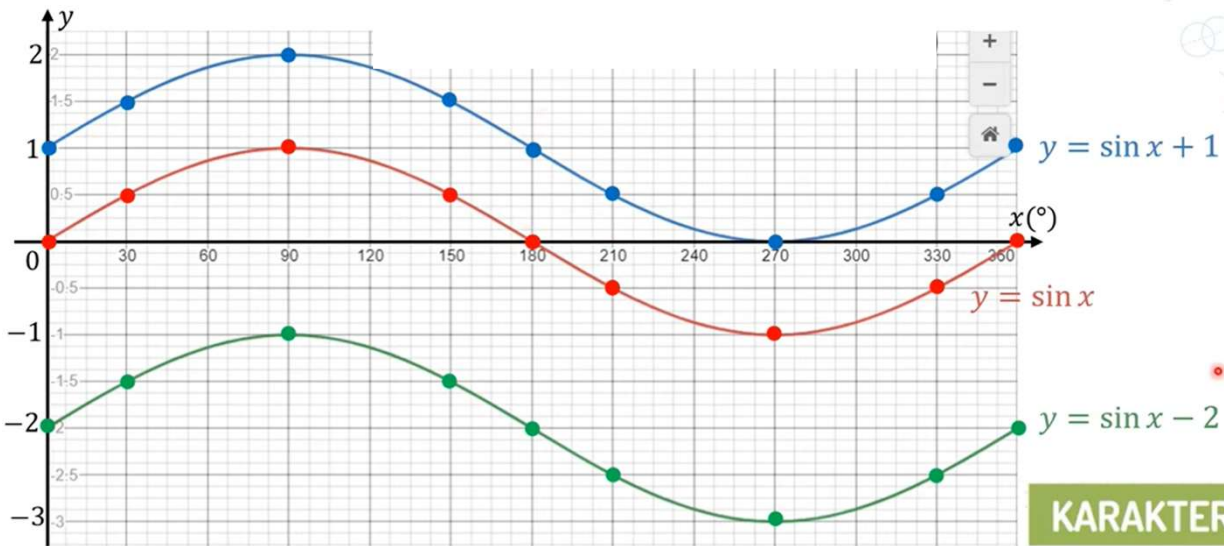


KARAKTERISTIK	$y = \sin x$	$y = \sin(x - 30^\circ)$	$y = \sin(x + 60^\circ)$
Nilai maksimum	1	1	1
Nilai minimum	-1	-1	-1
Amplitudo	1	1	1
Periode	360°	360°	360°



Fungsi Trigonometri (25)

□ Pergeseran Vertikal (D)



KARAKTERISTIK	$y = \sin x$	$y = \sin x + 1$	$y = \sin x - 2$
Nilai maksimum	1	2	-1
Nilai minimum	-1	0	-3
Amplitudo	1	1	1
Periode	360°	360°	360°

D



Fungsi Trigonometri (26)

Soal (5)

Gambarlah grafik dari $y = \sin 2x + 1$, untuk $0^\circ \leq x \leq 360^\circ$. Kemudian tentukan nilai maksimum, nilai minimum, amplitudo dan periodenya? Petunjuk urutan prioritas : $y = A \sin B(x + C) + D$

$$y = \sin(2x + 1)$$

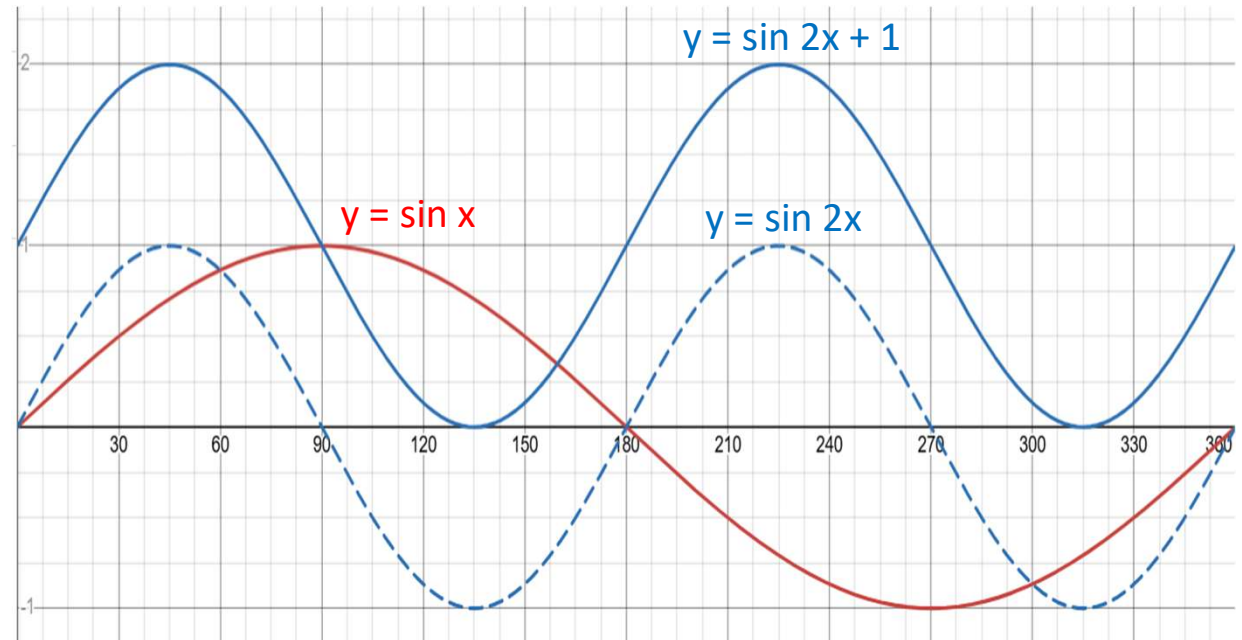
$$y = A \sin B(x + C) + D$$

3 2 1 4

Langkah 1: Buat grafik $y = \sin x$ (warna merah)

Langkah 2: Pada soal hanya ada nilai B (prioritas 2), yaitu 2. Artinya grafik $y = \sin x$ akan merapat menjadi $\frac{1}{2}$ dari amplitudo awal (awal 360° menjadi 180°) yaitu menjadi $y = \sin 2x$ (warna biru putus)

Langkah 3: Pada soal hanya ada nilai D (prioritas 4), yaitu 1. Artinya grafik $y = \sin 2x$ akan bergeser keatas (vertikal) sejauh 1 satuan dari titik awal dan menjadi $y = \sin 2x + 1$ (warna biru penuh)





DIKTISAINTEK
BERDAMPAK



TERIMA KASIH